Concepts to be Covered

- Circuit Theory (Using National Instruments Multisim* Software)
  - Kirchhoff’s Voltage and Current Laws
  - Voltage Divider Rule
  - Resistances - Series and Parallel Resistors
  - Capacitance
    - Series and Parallel Capacitors
    - Charging and Discharging a Capacitor
  - Advanced Topics
    - Inductors
    - Impedance (Z) and Admittance (Y)
    - Thévenin’s Theorem
    - Superposition Theorem

* Multisim - Simulation Program with Integrated Circuit Emphasis - SPICE

Circuit Theory – Ohm’s Law

- Ohm’s Law:
  - \[ V = IR \]
  - Where
    - \( V \) = Voltage in volts, V.
    - \( I \) = Current in amps, A.
    - \( R \) = Resistance in ohms, \( \Omega \).
  - For example, if the voltage is 5 V (volts direct current), and the resistor is 220 \( \Omega \), the current flow would be:
    \[ I = \frac{V}{R} = \frac{5}{220} \approx 0.0227 \text{ A} \text{ or } 22.7 \text{ mA} (\text{milliamperes}) \]
Ohm’s Law Simulation…

LRC meter showing .220 kΩ resistor.
(Inductance-Resistance-Capacitance)

Power supply set at 5 Vdc.
(Voltage regulated - Current limited)

Multimeter in series with the power supply and resistor,
reading ~22 mA.
(Volts AC & DC, Resistance, Diodes, Current)

Real World Measurement…

Kirchoff’s Voltage Law

- Kirchoff’s Voltage Law:
  - \( \sum V_{\text{source}} = \sum V_{\text{drop}} \)
  - The sum of the voltage drops in a series circuit
    equals the sum of the voltage sources.
  - \( V = V_1 + V_2 \)
- For series resistors, \( R_{\text{total}} = R_1 + R_2 + \ldots + R_n \)
- For example, if \( V = 5 \) V, \( R_1 = 1000 \) Ω, and \( R_2 = 220 \) Ω,
  then the voltage drop across each
  resistor is determined as follows:
  - Total resistance, \( R = 1220 \) Ω.
  - \( I = \frac{V}{R} = \frac{5 \text{ V}}{1220 \Omega} \approx 4.1 \text{ mA} \)
  - The voltage drop across \( R_1 \) is
    \( V_{R_1} = I R_1 = 0.0041 \times 1000 = 4.1 \text{ V} \)
  - The voltage drop across \( R_2 \) is
    \( V_{R_2} = I R_2 = 0.0041 \times 220 = 0.9 \text{ V} \)
  - Note that 4.1 V + 0.9 V = 5 V

Image courtesy of Texas Instruments
Kirchoff’s Current Law
• Kirchoff’s Current Law:
  - \( \sum I_{in} = \sum I_{out} \)
  - The sum of the currents entering a junction equals the sum of the currents leaving the junction.
  - \( I_1 + I_2 = I_3 + I_4 \)

Voltage Divider Rule
• Notice that the output voltage in this circuit is simply the voltage drop across \( R_2 \).
  - \( V_{out} = I R_2 = \frac{V}{R_T} R_2 = V \frac{R_2}{R_1 + R_2} \)
• Also notice that the output circuit will eventually provide additional pathways for the current to flow!
Parallel Resistors & Current Flow

- Notice that current will flow through both resistors (plus any future additional output circuitry).
- Total resistance for parallel resistors is as follows:
  \[ \frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2} \]
  \[ I = \frac{V}{R_{\text{total}}} \]
  \[ I_1 = \frac{V}{R_1} \text{ and } I_2 = \frac{V}{R_2} \]
  \[ I_f = I_1 + I_2 \]
- For example, if \( V = 5 \text{ V} \), \( R_1 = 1000 \Omega \) and \( R_2 = 2200 \Omega \), then:
  \[ \frac{1}{R_{\text{total}}} = \frac{1}{1000} + \frac{1}{2200} \approx 688 \Omega \]
  \[ I_1 = \frac{5}{1000} = 5 \text{ mA} \]
  \[ I_2 = \frac{5}{2200} = 2.27 \text{ mA} \]
  \[ I_f = 5 + 2.27 = 7.27 \text{ mA} \]
  \[ I_f = \frac{5}{688} \approx 7.27 \text{ mA} \]

Image courtesy of Texas Instruments
Example: Current Limiting an LED…

Resistor Color Code

Power…

- The power through a resistor:
  - \( P = I^2R = VI \) in watts.
- Resistors must dissipate heat, and are rated based on watts – typically \( 1/8, 1/4, 1/2 \) or a 1 watt resistor.
- The resistor may be made of carbon composition, carbon film, metal film, metal oxide film, foil or even wire wound.
Capacitance

- Capacitors are made of two parallel plates, separated by a dielectric material. These may be foil plates and rolled in a tubular shape or simply bonded as flat metal plates.
- The charge on a capacitor is: $Q = CV$, where
  - $Q$ is the charge in Coulombs,
  - $C$ is the capacitance in Farads and
  - $V$ is the voltage in Volts.

- $C = \varepsilon_0 \varepsilon_r \frac{A}{d}$, where
  - $C$ is the capacitance, in farads;
  - $A$ is the area of overlap of the two plates, in square meters;
  - $\varepsilon_r$ is the relative static permittivity (sometimes called the dielectric constant) of the material between the plates (for a vacuum, $\varepsilon_r = 1$);
  - $\varepsilon_0$ is the electric constant $\varepsilon_0 \approx 8.854 \times 10^{-12} \text{ F/m}$ - farads per meter;
  - $d$ is the separation between the plates, in meters;

RC Charging of a Capacitor...

- Note the exponential rise in voltage across the capacitor as it charges.
- The oscilloscope is set at 1V/div and 50ms/div.
- The voltage across the resistor and current flow both fall as the capacitor charges.

\[ V(t) = V_0 \left(1 - e^{-\frac{t}{RC}}\right) \]
\[ Q = CV_0 \left(1 - e^{-\frac{t}{RC}}\right) \]
\[ I = \frac{V_0}{R} e^{-\frac{t}{RC}} \]

Charging...

From previous example:
- $R=750 \ \Omega$
- $V_0=5 \ \text{Vdc}$
- $C=100 \ \mu\text{F}$
- $RC=0.075 \ \text{s}$
- (i.e. $t=2s$ is $2.7 \ \text{RC}$’s)
- $I_{max}=0.00667 \ \text{amps}$
- (Upon closing switch.)
- $Q_{max}=500 \ \mu\text{C}$
**RC Discharging of a Capacitor…**

- Note the exponential fall in voltage across the capacitor as it discharges.
- The oscilloscope is set at 1V/div and 50ms/div.
- At the onset of discharge, voltage across the resistor is the capacitor voltage, and then rapidly decreases along with the current.
  
  \[ V(t) = V_0 e^{-\frac{t}{RC}} \]

\[ Q = CV_0 e^{-\frac{t}{RC}} \]

\[ I = \frac{V_0}{R} e^{-\frac{t}{RC}} \]

**Discharging…**

From the previous example:

- \( R = 750 \Omega \)
- \( V_0 = 5\text{Vdc} \)
- \( C = 100\mu\text{F} \)
- \( RC = 0.075\text{s} \)
- \( \text{t}_{\text{max}} = 0.00867 \text{amps} \)
- (Upon closing switch.)
- \( Q_{\text{max}} = 500\mu\text{C} \) (the starting charge)

**Parallel vs Series Capacitors**

- To calculate the equivalent capacitance of capacitors in parallel, use the following formula:
  \[ C_{\text{Total}} = C_1 + C_2 + \cdots + C_n \]

- For series capacitors use:
  \[ \frac{1}{C_{\text{Total}}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n} \]

- Notice the similarity to resistors in series and parallel respectively (just reversed).
Advanced Topic: Inductors or Chokes

- Current flowing through a wire-wound coil produces a magnetic flux.
- A secondary voltage is induced into the coil by the movement of the magnetic flux as it opposes or resists any changes in the electric current flowing through it.
- An inductor opposes the rate of change of current flowing through it.
- Inductance is the amount of voltage dropped across an inductor for a given rate of change of current flowing through it.
- The unit of inductance is the Henry (H).

Determinants of the Self-Induced Voltage...

\[ V_L = N \frac{d\Phi}{dt} = \mu N^2 A \frac{di}{dt} \]

Where:
- \( N \) is the number of turns,
- \( A \) is the cross section in \( \text{m}^2 \),
- \( \Phi \) (phi) is the amount of flux in webers,
- \( \mu \) (mu) is the permeability of the core material
- \( l \) is the length of the coil in meters and
- \( \frac{di}{dt} \) is the rate of change of current in \( \text{A/S} \)

Calculating the Back-EMF...

\[ V_L(t) \frac{d\Phi}{dt} = \frac{dLI}{dt} = -L \frac{di}{dt} \]

Where:
- \( L \) is the self inductance and
- \( \frac{di}{dt} \) is the rate of change of current

- A circuit with an inductance of one H and current flow changing at a rate of one amp per second, will have an emf of one volt.
- With a dc current, \( \frac{di}{dt} = 0 \), and the induced emf voltage will be zero.
- With a steady dc current flowing, the inductor acts as a wire (short-circuit).
- Other concepts for independent study include power and energy storage in the magnetic field.
Consider series inductors in which there is no mutual inductance or magnetic coupling between the individual inductors.

Inductors will have a common current flowing.

The total inductance is the sum of the individual inductors:

\[ L_T = L_1 + L_2 + \cdots + L_n \]

For example, if \( L_1 = 20 \text{mH} \) and \( L_2 = 30 \text{H} \), then \( L_T = 50 \text{mH} \).

This is derived by calculating the sum of the individual voltage drops and dividing by \( \frac{di}{dt} \):

\[ L_T \frac{di}{dt} = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + \cdots + L_n \frac{di}{dt} \]

In the real world you need to allow for mutual inductance.

This may increase or decrease the total inductance depending on distance apart and orientation.

For cumulatively coupled coils:

\[ L_T \frac{di}{dt} = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + 2M \frac{di}{dt} \text{ or } L_T = L_1 + L_2 + 2M \]

(*minus 2M for differentially coupled inductors)
Example...

Consider two inductors in series, with inductance of 20mH and 60mH, and with measured total inductance of 100mH. What is the mutual inductance (M)?

- \( L_T = L_1 + L_2 + 2M \)
- \( 100 = 20 + 60 + 2M \), therefore
  \[ M = \frac{20}{2} = 10 \text{mH} \]

Parallel Inductors...

For parallel inductors, the total inductance can be calculated as follows:
\[
\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_n} \quad \text{or} \quad L_T = \frac{L_1L_2}{L_1+L_2} \quad \text{(if two inductors only)}
\]

Allowing for mutual inductance:

- \( L_T = \frac{L_1L_2-M^2}{L_1+L_2+2M} \) (mutually aiding)
- \( L_T = \frac{L_1L_2-M^2}{L_1+L_2+2M} \) (mutually opposing)

Example...

Consider two parallel inductors with values of 50mH and 25mH respectively, and mutually aiding. The mutual inductance is given as 11mH.

- \( L_T = \frac{L_1L_2-M^2}{L_1+L_2-2M} = \frac{50 \text{mH} \times 25 \text{mH} - 11 \text{mH}^2}{50 \text{mH} + 25 \text{mH} - 2 \times 11 \text{mH}} = 21.3 \text{mH} \)
Impedance

- **Impedance (Z)** – Total opposition a device or circuit offers to flow of an alternating current at a given frequency.
  - Units are in ohms
  - Complex quantity graphically shown on a vector plane (real and imaginary parts): 
    \[ R + jX \text{ where } R \text{ is resistance and } X \text{ reactance} \]

Admittance...

- **Admittance (Y)** – the reciprocal of impedance.
  - Units are in siemens (S)
  - Complex quantity (real and imaginary parts):
    \[ Y = \frac{1}{Z} = G + jB \]
    where \( G \) is conductance and \( B \) susceptance

Thevenin’s Theorem

- A means for isolating a portion of a circuit to simplify analysis, rather than solving loop and node equations.
- Used when the input source is a voltage (in contrast to Norton’s theorem which is used when the source is a current).
Applying Thevenin’s Theorem…

- How do we calculate the output voltage?
  - We can isolate the area in blue and consolidate it to a simple series circuit with a resistance, or impedance equivalent.
  - Start by calculating the output voltage at X and Y, considering only the blue shaded area. This is simply the voltage drop across \( R_2 \), or \( V_{out} = \frac{V}{R_2} \) (the voltage divider rule).

- Now short the voltage source and calculate the equivalent resistance, or \( R_{\text{eq}} = \frac{R_1 R_2}{R_1 + R_2} \) (our equation for parallel resistors).

- Finally, considering the circuit in red, applying the voltage divider rule again, we can calculate the output voltage:
  \[ V_{out} = \frac{V_{in} R_2}{R_1 + R_2} \]

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Superposition Theorem

- How do we calculate an output voltage when there are more than one voltage source?
  - When there are independent sources, the voltages and currents resulting from each source can be calculated separately, and the results are added algebraically.
  - Simply image all but one source as being shorted to ground, calculate the output and current and repeat for each source. The output voltage is then the sum of the individual outputs.

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Summary

  - Kirchhoff’s Voltage and Current Laws
  - Voltage Divider Rule
  - Resistances - Series and Parallel Resistors
  - Capacitance
    - Series and Parallel Capacitors
    - Charging and Discharging a Capacitor
  - Advanced Topics
    - Inductors
    - Impedance (Z) and Admittance (Y)
    - Thevenin’s Theorem
    - Superposition Theorem