WORK DEVELOPMENT IN THE MUSCLES OF THE JUMPING FROG

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Abstract
The energy expended as work by a frog when it jumps can be accurately and easily measured with the Laser Activated Amphibian Monitor System (LAAMS). The motion of the center of mass of a frog in flight is fully understood. This includes an analysis of air resistance and the resulting drag. Several equations are described that have application with both LAAMS and the former photographic techniques used to study frog jumping. These equations when used with LAAMS require computer manipulation. The required computer program is described in detail.

The advantage of LAAMS over former techniques and its potential as an investigative tool will become evident as our story of work development leaps forward!

## Introduction

Power development is defined as the total energy expended divided by the product of muscle mass and the time over which the energy was expended.

$$
P D=E T /(M M \times T A)
$$

In the frog MM is the mass of the muscles that contract in the hind limb when the frog jumps. The time of acceleration or $T A$ is the time interval beginning with the first sign of force against the ground to the instant the distal phalanges leave the ground.

The recently developed Laser Activated Amphibian Monitor System (LAAMS) allows for easy and rapid measurement of work development.

$$
W D=E W /(M M \times T A)
$$

The energy expended as work, EW, is the sum of kinetic energy and unrecovered potential energy expended by the frog as it jumps. As we will see later, power development and work development are closely related.

By forcing the frog to jump several times within a short time period, a work development decay curve can be constructed. In future experiments several frogs (chosen for similarity of certain characteristics) will be divided into control and experimental groups. Differences in work development decay curves found between groups (each frog in each group makes several jumps) may be attributed to effects of the experimental variable. Choice of the experimental variable is almost unlimited. For example, the effect of selected drugs on the functioning of the muscles might be monitored via work development measurements. Other variables include temperature, air composition, the frogs diet and even the sex and age of the frog. Moreover, differences in work development between cifferent species of frogs and toads can be observed, as well as seasonal and regional variations within a given species. An understanding of these "natural variables" is important if an accurate analysis of the effects of artificial variables is to be accomplished.

Power development and work development are related by way of the mechanical efficiency of muscle. The total energy spent by muscle during contraction is the sum of work and heat energy.

$$
E T=E W+E H
$$

The mechanical efficiency of muscle is the ratio of the work performed by muscle to the total energy spent (1).
efficiency = EW / ET

The total amount of heat liberated when muscle contracts has been the center of considerable controversy. Hill ( $2,3,4,5$ ) contends that when muscle contracts energy is liberated in three forms: 1.) A, heat of activation, 2.) heat of shortening, ax, and 3.) the mechanical work EW.

$$
\mathrm{ET}=\mathrm{EW}+\mathrm{A}+\mathrm{ax}
$$

The amount of shortening is $x$, and $a$ is a constant. In a tetanic contraction the heats of activation due to successive stimuli are together known as the maintenance heat. Carlson, Hardy and Wilkie (6) believe that shortening heat as such does not exist. Their technique for measuring total energy output is based on measurement of phosphocreatine hydrolysis. Phosphocreatine hydrolysis did show a constant term that corresponds to A above and a linear term that corresponds with EW. If the total quantity of heat liberated during a frog's jump could be measured, then an accurate power development figure would be obtained. Since heat can be measured in the isolated muscle, approximate values for muscle efficiency can be obtained. Multiplying the reciprocal of efficiency of the hind leg muscle system times the work development will give the power development.

$$
P D=(1 / \text { efficiency }) x \text { WD }
$$

The power development decay curve of a frog forced to jump several times consecutively is representative of the decrease in available chemical energy for muscle contraction. Once significant differences are obtained between a control group and an experimental group of frogs, the next task is to account for why the differences occur. The ultimate explanations center around variances in the normal biochemical processes of muscular contraction. An understanding of the "normal processes" has only recently come to light-and much remains to be explained. The mechanism of contraction has been carefully studied by Huxley (7).

The work of Maruyama and Weber (8) on the binding of ATP to myofibrils is representative of biochemical studies that are beginning to bring the process of contraction into focus.

## Previous Works on Frog Jumping

Although work development in the jumping frog has been previously unexplored, many attempts have been made to relate the distance an amphibian jumps to physical dimensions of the animal. Rand and Rand (9,10) have determined that the total body length of Bufo Marinus varies with the cube root of the weight and with the hind limb length. Moreover, they have also shown that the distance jumped varies directly with the hind limb length. Stokely and Berberian (ll) were unable to conclude that "changes in the relative proportions of the skeletal parts of the frog hind leg appreciably influence jumping ability." Gans and Rosenberg (12) determined a linear relationship between body weight of Bufo Marinus and the estimated force exerted when the animal jumps. (Gans and Parsons (13) have given a rather interesting analysis on the origins of the jumping mechanism in frogs).

Gray's work on animal locomotion, Hirsch's work on the mechanism of frog jumping, and E.C.B. Hall-Cragg's analysis of the jump of the LessGalago are perhaps the most useful works in regard to work development in the muscles of the jumping frog. Gray ( 14,15 ) in his discussion of amphibian jumping correctly describes the movement of the center of mass of the frog from the time it is in flight to the instant it touches the ground. His mathematical interpretation of the event is however extremely limited. Nevertheless, his photographs do clearly show the orientation of the limbs during flight. Hirsch (16) has done an excellent analysis of the acceleration phase. Figure 1 shows the form of the frogs acceleration. Figure 2 shows from a side and top view the orientation of the limbs during the various stages of figure l. The drawings in figure 1 were made by Hirsch via high speed photography. Former techniques used to measure the initial velocity of a jumping frog consisted of measuring the path over which the frog accelerated and the time spent accelerating. Unfortunately this is an extremely inaccurate procedure. The actual path taken by the center of mass can only be estimated. In addition, the acceleration is not uniform and leads to complications when trying to determine the initial velocity. Nevertheless, photographs do give an accurate


Fig. 1. The spring-like thrust of acceleration is easily seen in Hirsch's high-speed photographs of the jump.


Fig. 2. These side and aerial views of the bones and joints during the acceleration phase correspond with the individual frames of figure 1. The corresponding bones from the pelvic girdle downward are the femur, tibio-fibula, astragalus/calcaneum, metatarsal and phalanges. (A complete discussion on the bones and muscles of the hind limb can be found in Dr. Alexander Ecker's The Anatomy of the Frog, Oxford, 1889).
measure of the angle of trajectory. Developing photographs for each jump of an entire experiment consisting of many frogs and many jumps would be expensive and time consuming. LAAMS is capable of electronically measuring all the parameters of an amphibian jump on a real time basis. No assumptions are made about the acceleration, and air resistance may be included into the calculations.

Hirsch's work paired with Zugs (17) work on the relation between jumping and osteometrics may eventually allow for calculation of the change in length individual muscles undergo during the acceleration phase. Given work development of a particular jump the technique used by E.C.B. Hall-Craggs can be used to determine the force and tension developed in individual muscles. This technique consists of determining the force each muscle must apply to given joints so that the resultant of all forces produces the final measured force. Given Hirsch's side and top views of the limb orientation as acceleration proceeds, the angular velocity with which the various joints "open up" may be calculated. This would allow for calculation of the velocity of shortening of individual muscles. Admittedly the above argument needs considerable modification and evaluation before ever being attempted. It is presented however to show that energy relationships in the living animal should not be passed off as impossible items to measure. If velocity, tension, and length of individual muscles in the accelerating frog are determined, then it may be possible to construct a three-dimensional representation of the chemical energy available for contraction. (18)

LAAMS

The Laser Activated Amphibian Monitor System is designed to measure the time spent accelerating, time in flight, and the final horizontal displacement of the center of mass of a jumping frog. This data paired with measurements taken from a simple center of mass exercise can be used to generate the initial velocity ( $V_{O}$ ) and initial angle of takeoff ( $\theta_{0}$ ). In addition, as seen in figures 7, 8 and 9, the following information can be calculated:

1) $\theta \mathrm{w}=$ reentry angle
2) $V_{W}=$ reentry velocity
3) Most of the labeled segments (some are measured)
4) Potential energy
5) Kinetic energy
6) Work development
7) Approximate power development
8) Drag

An explanation of the various segments, points and other parameters, as well as the necessary equations for their solution will be discussed in the next section.

The LAAMS is best described by the flowchart of figure 3. The system consists of three main units--a large, long and narrow plexaglass tank (figure 4), a automatic temperature controller that maintains the tank air temperature at a predetermined level ( $10^{\circ} \mathrm{C}-50^{\circ}$ ), and a photocell decoder.

The three units interact in the following way. An experiment is selected and the necessary frogs are obtained. The tank's atmospheric composition, temperature, and lighting are then set to conform with the experiment's requirements. (atm. pressure must not deviate by more than approximately 25 mm Hg from the external pressure). A frog is chosen and placed inside the tank on top of the platform shown in figure 4. This platform is in reality a tread that will automatically bring the frog to the "zero line" whenever the frog is at rest and away from the zero line. In other words, after the frog jumps it is brought back to the beginning point automatically.

The laser provides two planes of light that are planar with the platform. One is located even with the tread and is normally blocked or "off" when the frog is at rest. The other plane is located just above the frog's head (it is adjustable) and is normally "on" when the frog is at rest. The planes are in reality each composed of a continuous laser beam that is bounced back and forth on end mirrors, giving in effect "planes" of light. A top and side view of this arrangement is shown in figure 5. The two starting beams for each plane are derived by splitting a single $\mathrm{He}-\mathrm{Ne}$ laser beam. The beams, after being bounced back and forth on the mirrors, land on their own photodiode. Therefore, any interruption of the "planes"


Fig. 3. The Laser Activated Amphibian Monitor System (LAAMS). Work development in the muscles of the jumping frog (or toad) can be measured automatically while the environment in which the frog is jumping can be controlled with regards to temperature, noise levels, lighting, humidity, and air composition. The laser used in the system is the Bausch and Lomb Helium-Neon Gas Laser. (. 1 mm multimode). (Safety precautions necessary when using lasers in the laboratory can be found in the Handbook of Laboratory Safety, CRC 1967). Additional information on the system can be found in "LAAMS: Design, Circuit Theory and Construction Techniques" by Steven Saliterman, 1973.


Fig. 4. Sketch of the tank used in LAAMS.
causes a corresponding loss of illumination of the corresponding photodiode. The lower photodiode is labeled DA, the upper photodiode DB. In addition to these two light receivers, a third photodiode (DC) is located at the "zero line" at tread level and is illuminated by a source shining perpendicular to the direction of tread travel. Table 1 shows the activity of the frog as it relates to the state of the individual light sensors.

Activity
at rest
accelerating
in flight
decelerating
being returned

| $\frac{\text { Photodiode }}{}$ |  |
| :--- | :--- |
| $\frac{D A}{\text { off }} \frac{D B}{\text { on }}$ | $\frac{D C}{\text { off }}$ |
| off off | on/off |
| on off/on | on |
| off off | on |
| off on | on |

Photodecoder Voltage
1 volt
3 volts
4 volts
3 volts
2 volts

Table l. eg. When the frog is accelerating, both planes are blocked and photodiodes A, B, and C are "off". The output voltage of the photodecoder in 3 volts. (explained later). When the frog first begins to accelerate its head rises and blocks beam DB. When acceleration ends and flight begins, DA is instantly turned on again. (DB may be either on or off during the flight phase). Deceleration begins and flight ends when the frog first touches the tread again. Thiss is detected by DA suddenly going from on to off. Deceleration ends when the frog is at rest and $D B$ is once again on. This position-rest away from the "zero line"-is detected by DC being on. The tread is automatically activated and the frog is brought back to the "zero line".

The three photodiodes signals enter (via amplifiers) a photocell decoder device. This device translates the binary type code word of the three photodiodes into four discrete output channels--each representing a given activity of the frog. (acceleration and deceleration activate the same channel). These channels (if desired) may each be connected to an individual electronic timer that will record the length of time each channel is "activated". The timers can then be connected via a memory unit to a teletype output device. The method currently being used is to voltage divide the 4 volt "activated" signal of each channel so that each channel produces a different voltage (see table l). These outputs can then be tied together and fed into an oscilloscope and paper recorder.


Fig. 5. Position of the upper and lower "planes" of laser light. The lower beam is blocked by the frog while the upper beam (positioned just above the head) is unobstructed. Photodiode DA is"off" while photodiode DB is "on". A third beam crosses the tread at the zero-line. When the frog is at rest at the zero-line photodiode DC is off. When the frog accelerates both DA and DB are off (DC is ignored). Flight is indicated when DA goes on: (DB is ignored), and terminated when DA goes off. During deceleration both DA and DB are off. When the frog is again at rest $D A$ is off while $D B$ and $D C$ are on. This combination causes the tread to automatically start. The tread stops when the frog is back at the zero-line and DC is off.

The output voltage will with time produce the graph seen in figure 6 . The oscilloscope sweep rate is set so that it can record the acceleration time, flight time and deceleration time in a single sweep. The recorder measures the return time. Several scope traces can be recorded on a single picture by slightly advancing the horizontal adjust after each jump.

Test runs have shown that the frog can be made to jump by a mild current-limited electric shock. When external lights are off and the inside of the tank illuminated with a blue light, the frog will jump down the lenght of the tank without hitting the sides. The preference for blue light by frogs has been demonstrated by Muntz (19) and Torelle (20).

The data necessary for future calculations is obtained in the following way. Time of flight and acceleration are read off of the photographs from the oscilloscope camera. The return time is determined by the lenght of the trace on the paper recorder. This time is then multiplied by the tread's velocity so that $B^{l}$ is obtained. The tread is activated by a rapid start motor and little time is lost in coming to a uniform velocity.

$$
B^{1}=\text { Return Time } x \text { Tread Velocity }
$$

The frog may jump before being fully returned as well as jump so far that it hits the end wall. In either case data from that jump must be discarded and indication of the "bad" data should be made on the paper recorder. Experience has shown that the frog quickly learns to jump only when it must - and the electric shock appears to be sufficient reason to jump. The sudden starting of the tread does not seem to cause the frog to jump. Future experiments might consist of a series of ten shocks spaced a minute apart. The interval between shocks is arbitrary and is chosen so as to produce the most desirable work development decay curve. Once the interval is chosen it must be maintained throughout the experiment. If the interval is to long the frog may fully recover from the previous jump and no decay curve would be obtained. If the interval is to short the frog may quit responding to the mild shock.


Fig. 6. Output voltage of the photodecoder. A different voltage corresponds with each activity of the frog. The signal above was selected so that the output would "somewhat" resemble a jump!


Fig. 7. Stages of the frog's jump and pathway taken by the center of mass during flight. Notice that the center of mass during flight travels further coming down from the summit of the trajectory than going up to the summit.

Mathematical Analysis of Frog Jumping

The following definitions correspond with the points and segments of figures 8 and 9:

Pl Hind end of frog before jump
P2 Tips of toes before jump
P3 Center of mass before jump
P4 Center of mass at instant of take off
P5 Intersection with x axis of vertical segment from P 4
P6 Tips of fingers before jump
P7 Intersection with path of c.o.m. of a horizontal segment from P4
P8 Intersection of a vertical segment through P7 with a horizontal through P9
P9 Center of mass at instant of landing
Pl0 Intersection with x axis of a vertical segment from P9
Pll Tips of fingers after landing
Pl2 Hind end of frog after landing
Pl3 Center of mass after landing
Pl4 Tips of toes after landing
P15 Intersection with x axis of a vertical segment from P3
P16 $1 / 2$ the distance from P4 to P7
P17 Highest point reached by the center of mass (summit)

A P2-Pll A=B+D
$B^{1}$ Pl-Pl2 Measured by LAAMS
B $\quad \mathrm{P} 6-\mathrm{P} 11 \quad \mathrm{~B}=\mathrm{B}^{1}$
C P5-P10 $\mathrm{C}=\mathrm{A}-\mathrm{O}-\mathrm{N}$
D P2-P6 Measured before jump
E P4-P7 $\quad E=\left(V_{0}^{2} \sin (2 \theta)\right) / g$
F P3-P4 discussed later
G P2-P3 discussed later
H P2-P4 Measured before jump
I $\mathrm{P} 5-\mathrm{P} 4 \quad \mathrm{I}=\mathrm{H} \sin \theta_{0}$
$J \quad \mathrm{P} 7-\mathrm{P} 8 \quad \mathrm{~J}=-\left(\mathrm{t}_{\mathrm{f}} \mathrm{V}_{\mathrm{O}} \sin \theta_{0}-.5 \mathrm{~g} t_{f}^{2}\right)$
$K \quad \mathrm{~K} 8-\mathrm{P} 9 \quad \mathrm{~K}=\mathrm{t}_{\mathrm{f}} \mathrm{V}_{\mathrm{O}} \cos \theta_{\mathrm{O}}-\left(2 \mathrm{~V}_{\mathrm{O}}^{2} \sin \theta_{\mathrm{O}} \cos \theta_{0}\right) / \mathrm{g}$


Fig. 8. In this figure $B^{1}$ and time of flight from P4 to P9 are determined by LAAMS. The points are explained in the section "Mathematical Analysis of Frog Jumping". Time of acceleration is also determined by LAAMS .


Fig. 9. The frogs have been removed so that each segment of figure 8 can be labeled. A sìmple center of mass exercise is performed for determination of $H, M, D$, and $Q$ for the particular frog being tested. These four variables along with $t_{f}, t_{a}, B^{l}$, mass of the frog and mass of the muscle are used for all other calculations.


Segments $E, J, K$ and $S$, and $V_{W}$ and $\theta_{W}$ are determined in the following way:
1.) The $x$ and $y$ components of $V_{o}$ are
$V_{o x}=V_{0} \cos \theta_{0} \quad V_{o y}=V_{o} \sin \theta_{0}$
The components of $V$ are
$V_{x}=V_{0} \cos \theta_{0} \quad V_{y}=V_{0} \sin \theta_{0}-g t$
The magnitude of the resulting vector at any instant is $\mathrm{V}=\left(\mathrm{V}_{\mathrm{x}}^{2}+\mathrm{V}_{\mathrm{y}}^{2}\right)^{l / 2}$
The angle $\Theta$ makes with the horizontal at that instant is $\theta=\tan ^{-1}\left(V_{y} / V_{x}\right)$
2.) The $x$-coordinate of the center of mass at any time is
$x=\left(V_{0} \cos \theta_{0}\right) t$
The $y$-coordinate is
$y=\left(V_{0} \sin \theta_{0}\right) t-.5 g t^{2}$
3.) The general equation for the parabolic trajectory is
$y=\left(\tan \theta_{0}\right) x-g /\left(2 V_{0} \cos \theta_{o}\right)^{2}$
4.) $\theta_{W}=\tan ^{-1}\left(\left(V_{0} \sin \theta_{0}-g t_{f}\right) /\left(V_{0} \cos \theta_{0}\right)\right)$
$V_{W}=\left(\left(V_{0} \cos \theta_{0}\right)^{2}+\left(V_{0} \sin \theta_{0}-g t_{f}\right)^{2}\right)^{0} 1 / 2$
5.) $t_{f}=$ time of travel of center of mass from $P 4$ to $P 9$ Let $t_{e}=$ time of travel of center of mass from P4 to P7
If an additional equal coordinate system is applied to figure 9 with origin at P4, then the $\mathrm{y}^{\prime}$-coordinate at P7 must equal zero. $y^{\prime}=0=\left(V_{0} \sin \theta_{0}\right) t_{e}-.5 g t_{e}^{2}$ Therefore $t_{e}=\left(2 V_{o} \sin \theta_{o} / g\right)$
6.) $E=t_{e} V_{o} \cos \theta_{0}=V_{o}^{2} \sin \left(2 \theta_{0}\right) / g$
7.) Let $t_{s}=$ time center of mass takes to travel from P4 to P17
$v_{0} \sin \theta_{0}=g t_{s}$
$t_{s}=\left(V_{0} \sin \theta_{0}\right) / g$
Let P17 $=\left(X_{17}^{\prime}, Y_{17}^{\prime}\right)$
$x_{17}^{\prime}=E / 2=v_{o}^{2} \sin \left(2 \theta_{0}\right) /(2 g)$
$Y_{17}^{r}=t_{S} V_{0} \sin \theta_{0}-\left(\mathrm{gt}_{\mathrm{s}}^{2}\right) / 2=\mathrm{V}_{0}^{2} \sin ^{2} \theta_{0} /(2 \mathrm{~g})$
Therefore $s=\left(V_{0}^{2} \sin ^{2} \theta_{0}\right) /(2 g)$
8.) $\mathrm{P} 7=\left(\mathrm{x}_{7}^{\prime}, \mathrm{Y}_{7}^{\prime}\right)=\left(\left(2 \mathrm{~V}_{0} \sin \theta_{0} / \mathrm{g}\right)\left(\mathrm{V}_{0} \cos \theta_{0}\right), 0\right)$
$P 9=\left(x_{9}^{\prime}, Y_{9}^{\prime}\right)=\left(t_{f} V_{o} \cos \theta_{o}, t_{f} V_{o} \sin \theta_{0}-.5 g t_{f}^{2}\right)$
$K=\left(x_{9}^{\prime}-x_{7}^{\prime}\right)=t_{f} V_{o} \cos \theta_{0}-\left(2 V_{0}^{2} \sin \theta_{0} \cos \theta_{0}\right) / g$
$J=\left(y_{7}^{\prime}-y_{9}^{\prime}\right)=-\left(t_{f} V_{o} \sin \theta_{o}-.5 g t_{f}^{2}\right)$
Segments $H, M$, and 2 are determined by a simple center of mass exercise performed with each frog. Figures 10 and 11 show the general procedure used for locating the center of mass. An anesthetized frog is tied together with light thread so that its limbs are in a position resembling that of rest. Two photographs are taken - each picture being of the frog suspended from a different point along the plane of symmetry. The photographs are then superimposed and the center of mass estimated by the intersection of the projected suspension lines. The distance from the center of mass to the usual ground is 2 . The distance from the tips of the toes to the tips of the fingers in the resting frog is easily measured while the frog is in the tank waiting to jump. Two determinations are made on the stretched frog.


Fig. 10. Determination of center of mass in the "resting frog" (see text for procedure).


Fig. 1l. Determination of center of mass in the "stretched" frog. Two determinations are made-one with the fore limbs back for measurement of $H$, the other with the fore limbs forward for measurement of $M$.

The first is done with the fore limbs tied back. This allows for measurement of $H$, the distance from the tips of the toes to the center of mass. The second determination is done with the fore limbs tied forward. This allows for determination of $M$, the distance from the tips of the fingers to the center of mass. In both determinations it is helpful to tie the frog to a thin rod placed placed down the lenght of the frog on its back. The suspension line is fastened to the center of the rod. The frog is then moved up or down the rod so that two pictures of different orientation can be taken.

At this point we can summarize the nine known variables.
$\mathrm{M}, \mathrm{H}$, and 2 - determined by center of mass exercise
D - measured while the frog is resting in the tank
$B^{1}, t_{f}, t_{a}$ - determined by LAAMS
$M_{f}, M_{m} \quad-m a s s$ of the frog and of the muscles involved in jumping (obtained by dissection)

The remaining segments of figure 9 , as well as $V_{w}$ and $\theta_{W}$ have been given as functions of these variables and $V_{0}$ and $\theta_{0}$. The next step is show that $V_{0}$ and $\theta_{0}$ are functions of the first six known variables above.

The following steps lead to three equations with three unknowns. The three unknowns are $V_{o}, \Theta_{0}$, and $\theta_{w}$. When all angles are kept between $1^{\circ}$ and $89^{\circ}$ inclusive, there is only one solution set.
1.) As before
$\theta_{W}=\tan ^{-1}\left(\left(V_{0} \sin \theta_{0}-g t_{f}\right) /\left(V_{0} \cos \theta_{0}\right)\right)$
I
2.) From figure 9
$\mathrm{J}=\mathrm{I}-\mathrm{L}$
$B=A-D=B^{1}$
$\mathrm{A}=\mathrm{C}+\mathrm{O}+\mathrm{N}$
$C=K+E$
3.) Therefore
$B^{1}=A-D=C+O+N-D=K+E+O+N-D$
$B^{1}=\left(t_{f} V_{O}+H\right) \cos \theta_{O}+M \cos \theta_{O}-D$

## 4.) As before

$$
J=-\left(t_{f} V_{0} \sin \theta_{0}-.5 g t_{f}^{2}\right)
$$

Therefore
$J=I-L=H \sin \theta_{O}-M \sin \theta_{W}=-\left(t_{f} V_{o} \sin \theta_{o}-.5 g t_{f}^{2}\right)$
Solving for $M$

$$
M=\left(H \sin \theta_{O}+t_{f} V_{o} \sin \theta_{O}-.5 g t_{f}^{2}\right) / \sin \theta_{W} \quad I I I
$$

In all of the equations $g$ is $980 \mathrm{~cm} / \mathrm{sec}^{2}$. The computer finds the solution set to equations I, II and III by means of a simple-routine. $\theta_{0}$ is incremented $1^{\circ}$ at a time. For each $\theta_{0}, V_{0}$ is incremented $1 \mathrm{~cm} / \mathrm{sec}$. The limits over which $\theta_{0}$ and $V_{o}$ are incremented are determined by solving for approximate values for $\theta_{0}$ and $V_{0}$. The limits are set plus and minus some interval on either side of the approximate values. For each set of $\theta_{0}$ and $V_{o}$, the computer solves the right hand sides of equations II and III and compares the answers to the known left hand sides. The computer then selects the best solution set.

Little has been said so far about the pathway taken by the center of mass during acceleration and deceleration. As we have already seen this information is not necessary for calculation of $V_{0}$ and $\theta_{0}$. The curved path taken during acceleration can be visualized by inspecting Hirsch's drawings (figures l and 2). During deceleration the center of mass continues forward for a few moments and then drops down and backward. Pl3 is not involved in any of our calculations.

A very rough approximation of the initial velocity could be obtained by assuming uniform acceleration over the segment F. F is calculated from $H, G$ and the angle $P 3-P 2-P 15 . \vec{V}_{o}$ is then equal to $2 F / t_{a}$. This equation is not used in any form in this project.

Obtaining Work Development
The computer program listed here is capable of handling the data from twenty frogs making ten jumps each (smaller numbers may be used). The data from four frogs making two jumps each has been simulated and entered into the program (see data at end of computer program). Frog 1 has been labeled "control." Thirty variables are listed and under each the appropriate value for the particular jump. The nine known values entered into the program as data are circled. The remaining
variables are solved by the formulas already described and by the following:

$$
\begin{gathered}
\text { Potential Energy }=P E=M_{f} g(I-Q) \\
\text { Kinetic Energy }=K E=.5 M_{f} V_{O}^{2} \\
\text { Work Development }=(K E+P E) / t a \times M m \\
\text { Power Development }=5 \text { (Work Development) }
\end{gathered}
$$

(where efficiency of the muscle has been arbitrarily set at 20\%)

$$
\begin{aligned}
& \text { Approximate } V_{O}=\left(B^{1}\right)^{2} / t_{f}^{2}+\left(t_{f}^{2} g^{2} / 4\right) \\
& \text { Approximate } \theta_{O}=\sin ^{-1}\left(t_{f} g\right) /\left(2 V_{O}\right)
\end{aligned}
$$

These approximate values are used only to define a region of $V_{0}$ and $\theta_{0}$ that the computer can use to rapidly find the real $V_{O}$ and $\theta_{O}$. The values in the output labeled F1 and F2 are the final solutions to equations II and III reached by the computer. These values agree fairly well with the actual $B^{l}$ and $M$, and indicate that $V_{O}$ is known to the nearest $\mathrm{cm} / \mathrm{sec}$ and $\theta_{0}$ is known to the nearest degree. (this has been varified by letting the computer print out an entire family of solns. as $V_{o}$ and $\theta_{0}$ are incremented)

In summary, inserting the nine circled variables allows for determination of work development. The other values determined are just nice things to have around. The next step in an experiment would be to graph work development against jump number for each frog. The control group data would then be statistically compared to the experimental group in hopes of finding significant differences.

A Final Note on Air Resistance
Up to now we have neglected air resistance and made our calculations as if the frog were in a vacuum. As we will soon see the effect of drag has almost no consequences on the determination of $\mathrm{V}_{0}$ and $\theta_{0}$. Equations I,II,III described earlier may be modified to conform with the additional force of $\mathrm{drag}, \mathrm{F}_{\mathrm{d}}$.
I. $\theta_{W}=\tan ^{-1}\left(\left(V_{O} \sin \theta_{O}-(s i n \theta / M f) \int_{t=0}^{t f} F_{d} d t-t_{f g}\right) /\left(V_{O} \cos \theta_{O}-\left(\cos \theta / M_{f}\right) x\right.\right.$ $\left.\left.\int_{t=0}^{t f} F_{d} d t\right)\right)$
II. $B^{\prime}=\left(t_{f} V_{o} \cos \theta_{0}-\left(\left(.5 \cos \theta_{0} / M_{f}\right) \int_{t=0}^{t f} t^{2} F_{d} d t\right)\right)+H_{\cos } \theta_{0}+M \cos \theta_{W}-D$
III. $M=\left(H \sin \theta_{O}+t_{f} V_{O} \sin \theta_{O}-\left(.5 \sin \theta_{O} / M f\right) \int_{t=O}^{t f} t^{2} F_{d} d t\right) / \sin \theta_{W}$

The problem now becomes one of solving the two integrals used above.


Fig. 12. General relationship between drag, velocity and time for a frog in flight.


Fig. 13. Reynolds number and coefficient of drag for a sphere, airship hull and disk. A sphere of diameter 4 cm . is used in the sample calculation. It should be realized that a frog in flight is more streamlined and suffers less drag than does the sphere of similar dimensions.

The general relationship between drag, time, and velocity of a jumping frog is shown in figure 12. The maximum drag occurs when the frog is traveling with its greatest velocity. The maximum velocity occurs at a time $t_{f^{--}}$when the frog has a final reentry velocity greater than its initial velocity $V_{o}$. The relationship between drag and time is arrived at by first determining the relationship between drag and velocity over the velocities concerned. This information along with the relationship between time and velocity allows the drag as a function of time to be determined. Once the drag as a function of time is known the integrals described above are easily solved.

For a sample calculation of the drag function we will use the simulation date of frog 1 in the computer output. The important data are $\mathrm{V}_{\mathrm{W}}=\mathrm{V}_{\text {maximum }}=189^{\circ}$, $\mathrm{V}_{\text {minimum }}=\mathrm{V}_{\mathrm{O}} \operatorname{Cos} \theta_{\mathrm{O}}=109 \mathrm{~cm} / \mathrm{sec}$, $\mathrm{V}_{0}=177 \mathrm{~cm} / \mathrm{sec}$ and $\theta_{0}=52^{\circ}$. Other necessary data include the viscosity, kinematic viscosity and density of dry air at 18 degrees centigrade at 1 atmosphere.

$$
\begin{gathered}
\text { viscosity }=\mu=182.7 \times 10^{-6} \quad \mathrm{~g} / \mathrm{sec}-\mathrm{cm} \\
\text { kinematic viscosity }=V=1.54 \times 10^{-1} \mathrm{~cm}^{2} / \mathrm{sec} \\
\text { density }=\rho=1.2 \times 10^{-3} \mathrm{~g} / \mathrm{cm}^{3}
\end{gathered}
$$

Using the Reynold number vs. coefficient of drag chart (fig. 13) table 2 can be constructed (2l). The frog is imagined as a sphere with diameter $d=4 \mathrm{~cm}$. "A" in table 2 is equal to $\pi(d / 2)^{2}$.

| V | $\underline{R}=\binom{$ V }{ d/v } | $\mathrm{C}_{\text {d }}$ | $F_{d}=\left(C_{a} \rho V^{2} A\right) 2$ |
| :---: | :---: | :---: | :---: |
| 177 | 4600 | . 4 | $380 \mathrm{~g}-\mathrm{cm} / \mathrm{sec}^{2}$ |
| 180 | 4680 | . 4 | 392 |
| 183 | 4750 | . 4 | 406 |
| 186 | 4830 | . 4 | 418 |
| 189 | 4900 | . 4 | 431 |

Table 2. Calculation of $d r a g$ for various velocities from $V_{\text {maximum }}$ to $V_{\text {minimum }}$ for frog 1. These results are graphed in figure 14.

Velocity is related to time by $\left.V=\left(V_{0} \operatorname{Cos} \theta_{0}\right)^{2}+\left(V_{o} \operatorname{SIN} \theta_{0}-g t\right)^{2}\right)^{1 / 2}$. The simulation data of frog 1 gives a time of flight $t_{f}=.3 \mathrm{sec}$. Intervals of .05 sec are graphed against the resulting velocity in figure 15 . Table 3 gives the values used.

| $t=$ | 0 | .05 | .1 | .15 | .2 | .25 | .3 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $v=$ | 177 | 140 | 115 | 109 | 121 | 151 | 189 |

Table 3. Relationship between velocity and time, from simulation data of


Fig. 14. Drag plotted against velocity.


Fig. 15. Velocity plotted against time.


Fig. 16. Drag plotted against time.
frog 1.

It is now possible to find the relationship between drag and time. Because the coefficient of $d r a g, C_{d}$, is constant over the velocities involved, $F_{d}$ is proportional to $\mathrm{V}^{2}$. Inspection of table 2 shows that $F_{d}=1.2 l x 10^{-2} \mathrm{~V}^{2}$. Replacing V by $\left(\left(\mathrm{V}_{\mathrm{o}} \operatorname{COS} \theta_{0}\right)^{2}+\left(\mathrm{V}_{\mathrm{o}} \operatorname{SIN} \theta_{0}-\mathrm{gt}\right)^{2}\right)^{1 / 2}$ gives the following function for $\mathrm{F}_{\mathrm{d}}$ :
$F_{d}=3.69 \times 10^{2}-3.32 \times 10^{3} t+1.16 \times 10^{4} t^{2}=a-b t+c t^{2}$ and $\int_{t=0}^{t_{f}} f_{d} d t=$


The insertion of these integrals into the modified equations described at the beginning of this section do not significantly alter the final determination of $V_{O}$ and $\theta_{0}$. Using the values for $a, b$ and $c$ in the example above does not result in a new $\mathrm{V}_{\mathrm{O}}$ and $\theta_{\mathrm{O}}$ when $\mathrm{V}_{\mathrm{O}}$ is measure to the nearest $\mathrm{cm} / \mathrm{sec}$ and $\theta_{0}$ measured to the nearest degree. Drag vs. time for the above example is graphed in figure 16. Rapid inspection shows that for a loog frog the max. acceleration experienced opposite to the direction of motion is about $4 \mathrm{~cm} / \mathrm{sec}^{2}$. This has a small influence when compared to the $980 \mathrm{~cm} / \mathrm{sec}^{2}$ downward acceleration due to gravity. Strictly speaking however, the drag causes a path of travel of the center of mass that is not exactly a parabola.

The values used for $a, b$ and $c$ in the above example were derived by using the $V_{o}$ as if in a vacuum. This allowed the two integrals and a new $V_{O}$ and $\theta_{0}$ to be calculated. In reality we have only found the first approximation to the real $\mathrm{V}_{\mathrm{O}}$ and $\theta_{0}$. The next step would be use the new $V_{0}$ and $\theta_{0}$ to calculate $a$ better set of values for $a, b$ and $c$. The actual values for $V_{0}$ and $\theta_{0}$ would be arrived at by successive approximation. Fortunately, not even the first calculation is necessary for this experiment!

## Program Operating Structure

The following information includes all the necessary information for running the program on the CDC 3300.
\$JOB, ID, NAME, 2,2000
\$SCHED, CORE=60, SCR=3, CLASS=C
\$FTN (L, X,M)
(Program)
END (column 7)

FINIS (column 10)
\$ X, LGO
(DATA)
88
Data is in the following format
Card 1 Any alphanumeric comment columns 2-80
Card 2 Any alphanumeric comment columns 2-80
Card 3 Total number of frogs. Two digit integer value in columns
1 and 2. (must be 20 or less).
Card 4 Frog characteristic card

| Variable | Starting Column | Format |
| :--- | :---: | :--- |
| Frog I.D. | 1 | I3 |
| Control/Exper. | 6 | A8 |
| Number of jumps | 16 | N2 (max. no. is 10) |
| Mass of frog (g) | 20 | F 6.2 |
| H (cm) | 28 | F 5.2 |
| M (cm) | F 5.2 |  |
| D (cm) | 35 | F 4.2 |
| Mass of muscle (g) | 48 | F 6.2 |
| Q | 56 | F 4.2 |

Card 5 Jump Data for first jump.
Time of flight (sec.) 5 F 4.2
Time of acceleration

| (sec) | 11 | F 4.2 |
| :--- | :--- | :--- |
| B $^{\prime}(\mathrm{cm})$ | 17 | F 6.2 |

Card 6-n Remainder of jump data for frog identified in card 4. The jump data cards are arranged in the same order as the jumps were made. One card is used for each jump.

Card n+l-last cart
Repeat as in card 4 for next frog. Place jump data immediately after frog characteristic card. Follow this procedure until all frogs and all jumps are included.

Design of the Computer Program
The following letters and descriptions refer to sectioned blocks labeled on the computer program.
A. Storage allocation of arrays
B. Input and output of data cards one and two.
C. Input of total number of frogs
D. Input of characteristics of an individual frog.

Identification number, control/experi., total number of jumps by this frog, mass of the frog, M, D, mass of the muscles, Q.
E. Input of jump data

Time of flight, time of acceleration, $B$
F. Acceptance range used later in comparing $F 1$ and $F 2$ to the real $B$ and $M$.
G. This loop increments frog being analyzed
H. This loop increments jump number of particular frog being analyzed
I. Used as counter
J. Assignment of array variables to "easier to handle" variables.
K. Determination of an approximate $V_{O}$ and approximate $\theta_{0}$ used to determine the limits over which a search for the actual $\mathrm{V}_{\mathrm{O}}$ and $\theta_{\mathrm{O}}$ will be conducted.
L. Loop increments $\theta_{0}$ being tested in equations $I, I I$, and III.

M Loop increments $V_{0}$ being tested in equations I,II, and III.
N. Equation I.
O. Equation II.
P. Equation III.
Q. If a soln. to equations $I, I I$ and $I I I$ is reached with the accuracy set by ( $F$ ) above, this block is reached. The $V_{O}, \theta_{O}, \theta_{W}, F_{1}$ and $F_{2}$ that are currently being used are stored for later evaluation. The computer than returns and searches for more possible solutions.
R. After all possible solutions have been reached, the computer here selects the solution in which the sum of the absolute errors of $\mathrm{F}_{1}$ to $B$ and $F_{2}$ to $M$ is smallest. Once the solution has been obtained, the correct $V_{O}, \theta_{O}, \theta_{W}$ and final $F_{1}$ and $F_{2}$ are save for use in (S). Calculation of other dimensions associated with the jump, and storage in "w" arrays. These "w" arrays are eventually printed out.

| $\mathrm{wl}=\mathrm{V}_{0}$ | w7=PE | wwl $=\mathrm{F}_{1}$ | ww $7=L$ | wwl3=K |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{w} 2=\theta_{0}$ | w8=KE | $\mathrm{ww} 2=\mathrm{F}_{2}$ | $w w 8=I$ | ww14=S |
| $w 3=V$ | w9 =WD | $w w 3=B$ | $w w 9=J$ | Ww15=AVO |
| ${ }^{W} 4=\theta_{\text {w }}$ | $\mathrm{w} 10=\mathrm{PP}$ | $w \mathrm{w} 4=\mathrm{M}$ | wwl0=A | ww16=CATO |
| w5=Q | wll= $\mathrm{M}_{\mathrm{f}}$ | $\mathrm{ww}^{5} 5=\mathrm{T}_{\mathrm{f}}$ | wwll=E |  |
| w6=I+S | $\mathrm{wl2}=\mathrm{Mm}$ | $\mathrm{ww}^{\text {w }}=\mathrm{T}_{\mathrm{a}}$ | wwl2=C |  |

T. Return to next jump or to next frog
U. Print out of calculations

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FACILITIES NOT USED
CORE $=006$
SCR $=000$
LINE $=1691$
$C A R D=0$
SUM OF OUT+PLT BLOCKS $=15$

JOB, GRAD29,SSS,2,2000
SCHED, CORE $=60, S C R=3$, CLASS $=0$
FTN (L, X,M)

MS FORTRAN 3.2/OHCS VER 5-7.

## PROGRAM FROGJUMP

OIMENSION SC1 (20), SC2(20), NSID(20), SGG(20), SMASS (20), SH(20), SM(20)-
$1, S D(20), N S T N J(20), \operatorname{SMASSA}(20), \operatorname{STF}(20,10), S T A(20,10), \operatorname{SG}(20,10)$
1,SF1(20), SF2(20),
$1 W 1(20,10), W 2(20,10), W 3(20,10), W 4(20,10), W 5(20,10), W 6(20,10)$,
$1 W 7(20,10), W 8(20,10), W 9(20,10), W 10(20,10), W 11(20,10), W 12(20,10)$
$1, W W 1(20,10)$, WW2 $(20,10)$, WW3 $(20,10)$, WW4 $(20,10)$, WW5 $(20,10)$,
1 WW6 $(20,10)$, WW $(20,10), W W 8(20,10)$,WWG(20, 10$), W W 10(20,10)$,
$1 W W 11(20,10)$, WW12 $(20,10)$, WW13 $(20,10), W W 14(20,10)$, WW15 $(20,10)$,
1 WW1 $(20,10)$, WWW1 $(20,12)$, WWW2 $(20,10), S F 3(20), S F 4(20)$
1,TT(20)
$1, S B(20,10)$
1, WWW3 (20,10), WWW4 (20,10), SF5 (20), SF6 (20)
1,SQ(20)-
INTEGER TNF
REAC $(60,1)(S C 1(K A), K A=1,10)$
$\operatorname{REAC}(60,1) \quad(S C 2(K B), K B=1,10)$
FORMAT(10A8)
WRITE $(61,3) \quad(S C 1(K C), K C=1,10)$
WRITE $(61,3) \quad(S C 2(K D), K D=1,10)$
3
FORMAT (//10A8)
READ (60, 33) TNF
33 FORMAT(I2)
DO $5 \mathrm{KE}=1$, TNF
PEAD (50, 44) NSID (KE), SGG(KE), NSTNJ (KE), SMASS (KE), SH(KE), SM (KE),
1 SO(KE), SMASSM(KE),SO(KF)
FORMAT (I $3,2 X, A 8,2 X, I 2,2 X, F 6.2,2 X, F 5.2,2 X, F 5.2,2 X, F 4.2,2 X, F 6.2$
1, 2X,F4.2)
INJ $6=$ NSTNJ $(K E)$
DO $5 \mathrm{KF}=1$, INJ6
READ (60, 6) STF (KE,KF), STA (KE,KF), SB(KE,KF)
CONTINUE
6 FORMAT( $4 \times, F 4,2,2 X, F 4.2,2 X, F 6.2)$
$\mathrm{G}=980$
$\mathrm{CC1}=.5$
$\mathrm{CC2}=.5$ F
DO $29 \mathrm{JF}=1$, TNF
INU5=NSTNU(JF)
D0 $29 \mathrm{JJ}=1$, INJ5 — H
$\mathrm{K}=0$
$T F=S T F(J F, J J)$
$B=S B(J F, J J)$
$H=S H(J F)$
$7 M=S M$ (JF)
$Z M F=S M A S S(J F)$
ZMM = SMASSM (JF)
$D=S O$ (JF)
AVo $=\left(\left(T F^{* *} 2^{*} G^{* *} 2\right) / 4+3^{* *} 2 / T F^{* *} 2\right) * * \cdot 5$
$X X=\left(T F^{*} G\right) /\left(2^{*} A \cup 0\right)$
$A \operatorname{TO}=\operatorname{ATAN}(x x /((1-(x x) * * 2) * * .5))$
Q1 $=A V 0+25$
D=AVO-100
$\operatorname{IF}(0 . L E \cdot C) Q=1$
CATO $=$ ATO* $350 /(? * 3.14)$

MS FORTRAN Z.?/DHCS VER 5-7.

```
    Q4=CATO +15
    Q3=CATO-15
    IF(Q3.LE.0)QE=1
    IF(04.GE.90) O4=89
    INJ1=0
    IN*2=01
    INJ3=03
    INJ4=04
    DO 10 J=INJ3,INJ4
    L
    xJ=J
    T0=(xJ*3.14*2.0)/360.0
    DO 10 IVO=INJ1,INJ2 ——M
    VO=IVO
    TN=ATAN(ABS ((VO*SIN(TO)-G*TF)/(VO*COS(TO))))——_N
    F1= (TF*VO+H)*\operatorname{cos}(TO)+ZM*}\operatorname{cos}(TW)-
        IF(ABS(B-F1).LE.CC1)7,9
    7 F2=(H*SIN(TO)+TF*VO*SIN(TO)-.5*G*TF*TF)/SIN(TW)-P
        IF(ABS(ZM-F2),LE.CC2) 8,9
    8 K=K+1
    TI(K)=(ABS(B-F1))/B+(ABS (ZM-F2))/ZM
    SF1(K)=F1
    SF2(K)=F2
    SE3(K)=VO
    SF4(K)=XJ
    SF5(K)=TW
    SFS(K)=T0
    9 CONTINUE
    10 CONTINUE
        DO 11 IY=1,K
        D0 101 JY=1,K
        IF(TT(IY).LE.TT(JY))GO T0 101
        GO TO 11
    101 CONTINUE
        WW1 (JF,JJ)=SF1(IY)
        WW2(JF,JJ)=SF2(IY)
        WWW1(JF,JJ)=SFZ(IY)
        WWW2(JF,JJ)=SF4(IY)
        WWW3(JF,JJ)=SF5(IY)
        WWW4(JF,JJ)=SFG(IY)
        GO TO 12
    11 CONTINUE
    12VO=WWW1(JF,JJ)
        XJ=WWW2(JF,JJ)
        TW=WWW3(JF,JJ)
        T0=WWW4(JF,JJ)
        W1(JF,JJ)=V0
        W2(JF,JJ)=xJ
        W3(JF,JJ) =((VO*COS(TO))**2+(VO*SIN(TO)-G*TF)**2)**.5
        W4(JF,JJ)=TW*36./(2*3.14)
        W5(JF,JJ)=SQ(JF)
Wo(JF,JJ) = H*SIN(TO) + ((VO**2)*SIN(TO)**2)/(2*G)
W7(JF,JJ)=ZMF*G* (H*SIN(TO)-SO(JF))
WB(JF,JJ)=.5*2MF*VO**2
W9(JF,JJ) = (W7 (JF,JJ) +W8(JF,JJ))/((ZMM)*STA(JF,JJ))
```

$\left.\begin{array}{ll}\text { W11 (JF, JJ) }=2 M F \\ \text { W12(JF, JJ) }=\text { IMM }\end{array}\right]$
$\qquad$

EXPERIMENT 1 SIMULATION


| KE Kinetic $E$. |  | $\begin{gathered} \text { WD } \\ \text { Work Development } \end{gathered}$ | PD Power Derelopment |  | (M7) <br> madss of frog | mass of muscte (D) | (0) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1.57E |  | 4.51 E 05 | 2.25E |  | 100.00 | 15.00 | 2.00 |
| 1.55 E |  | 4.73 E 05 | 2.36 E | 06 | 100.00 | 15.00 | 2.00 |
| A | E | c | K | S | Avo | CATO | (H) |
| A | E | c | $k$ | 5 | approvimate | $\underset{\theta 0}{\text { approyimate }}$ |  |
| 42.00 | 31.03 | 333.00 | 1.97 | 9.92 | 198.46 | 47.82 | 9.00 |
| $37.40 \quad 28$ | 28.42 | 229.66 | 1.24 | 11.36 | 191.90 | 54.82 | 9.00 |

FROG 2 CONTROL


FROG 3 EXPER.

| JUMP | VO | TO | VW | TW | 0 | $I+S$ | PE |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 188.00 | 49.00 | 188.40 | 49.11 | .98 | 16.30 | $4.90 E$ | 05 |
| 2 | 245.00 | 55.00 | 245.45 | 55.07 | .98 | 27.09 | $5.40 E$ | 05 |


| 1 | 44.27 | 7.88 | 44.01 | 8.01 | .29 | .33 | 6.05 | 6.04 | -0.01 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 2 | 64.82 | 7.85 | 65.00 | 8.03 | .41 | .33 | 6.56 | 6.55 | -0.01 |


| KE |  | WD | $P D$ |  | MF | MM | D |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1.75 E | 06 | 4.50E 05 | 2. 25 E |  | 99.00 | 15.10 | 2.01 |
| $2.97 E$ | 067 | 7.05E 05 | 3.52 E | 06 | 99.00 | 15.10 | 2.01 |
| A | $E$ | C | K | S | $A \cup 0$ | CATO | H |
| 46.01 | 35.72 | 35.52 | $-0.20$ | 10.26 | 207.88 | 43.15 | 8.00 |
| 67.01 | 57.58 | 57.83 | . 26 | 20.54 | 255.92 | 51.75 | 8.00 |


| JUMP | VO | TO | $V W$ | $T W$ | $Q$ | $I+S$ | PE |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 202.00 | 41.00 | 201.78 | 40.93 | 1.00 | 8.95 | $8.7 * F$ | 04 |
| 2 | 210.00 | 55.00 | 209.24 | 54.85 | 1.00 | 15.09 | $8.7 * E$ | 04 |


| UMMP | $B$ | $M$ | $B$ | $M$ | $T F$ | $T A$ | $L$ | $I$ | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 38.97 | .07 | 39.00 | 1 | .27 | .35 | 0 | 0 | 0 |
| 2 | 39.99 | .20 | 40.03 | 0 | .35 | .29 | 0 | 0 | 0 |


| KE | WD | PD | MF | MM | 0 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $1.82 E$ | 06 | $2.47 E$ | 05 | $1.23 E$ | 06 | 89.00 | 20.00 | 2.20 |  |
| $1.96 E$ | 06 | $3.23 E$ | 05 | $1.62 E$ | 06 | 89.00 | 20.00 | 2.20 |  |
| A |  |  |  |  |  |  |  |  |  |

