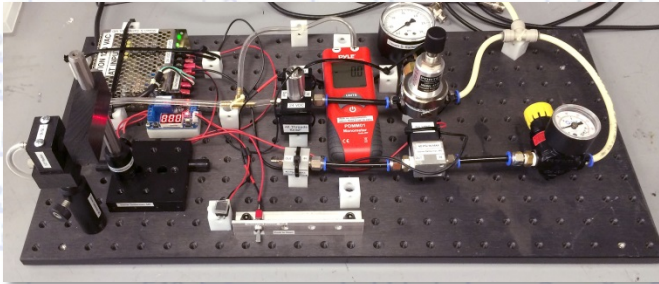# Introductory Medical Device Prototyping

Analog Circuits Part 1 – Circuit Theory

Prof. Steven S. Saliterman, http://saliterman.umn.edu/
Department of Biomedical Engineering, University of Minnesota
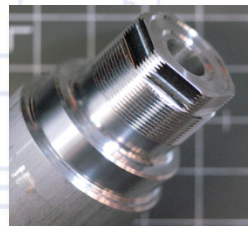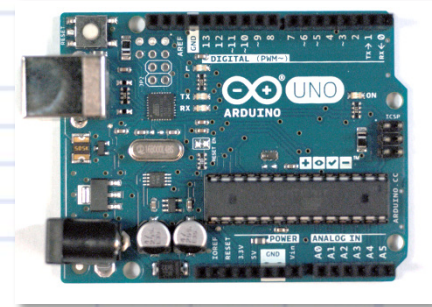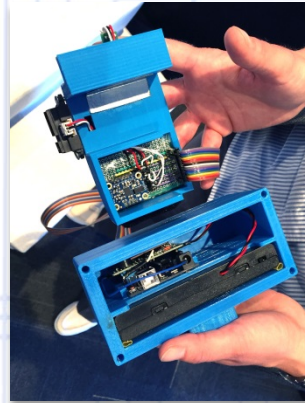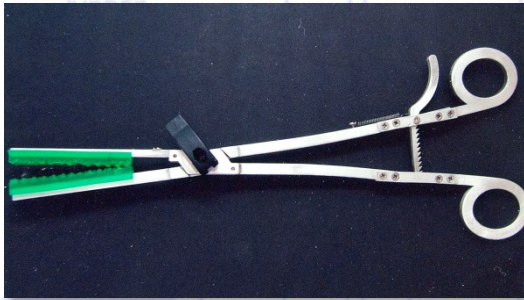
Concepts to be Covered

- Circuit Theory (Using National Instruments Multisim* Software)
  - Kirchoff's Voltage and Current Laws
  - Voltage Divider Rule
  - Resistances - Series and Parallel Resistors
  - Capacitance
    - Series and Parallel Capacitors
    - Charging and Discharging a Capacitor
  - *Self Review - Advanced Topics*
    - Inductors
    - Impedance (Z) and Admittance (Y)
    - Thevenin's Theorem
    - Superposition Theorem

* Multisim - Simulation Program with Integrated Circuit Emphasis - SPICE

Circuit Theory – Ohm's Law

- Ohm's Law:
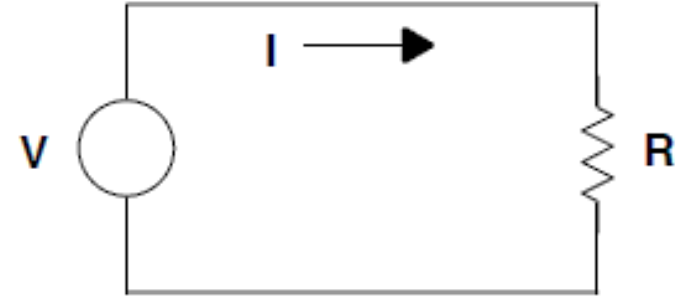 - $V = IR$
    - Where
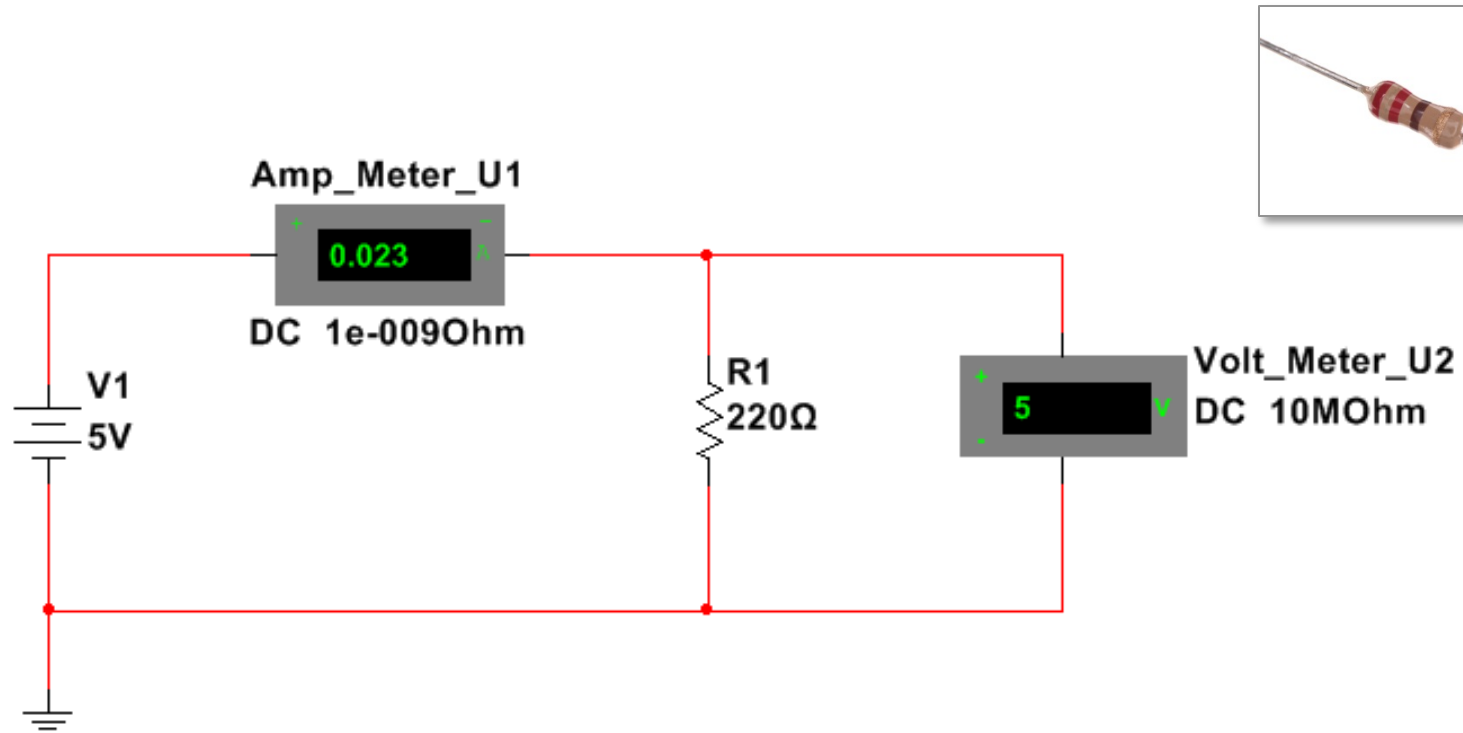   - V = Voltage in volts, V.
      - I = Current in amps, A.
      - R = Resistance in ohms, Ω.
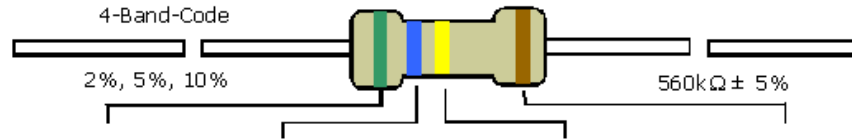 - For example, if the voltage is 5 V (volts direct current), and the resistor is 220 Ω, the current flow would be:

$$I = \frac{V}{R} = \frac{5}{220} \sim .0227 \, A \, or \, 22.7 \, mA \, (milliamps)$$

Image courtesy of Texas Instruments

Ohm's Law Simulation…

Resistor Color Code…

4-Band-Code

2%, 5%, 10%

560kΩ ± 5%

COLOR	1st BAND	2nd BAND	3rd BAND	MULTIPLIER	TOLERANCE	
Black	0	0	0	1Ω		
Brown	1	1	1	10Ω	± 1%	(F)
Red	2	2	2	100Ω	± 2%	(G)
Orange	3	3	3	1KΩ		
Yellow	4	4	4	10KΩ		
Green	5	5	5	100KΩ	±0.5%	(D)
Blue	6	6	6	1MΩ	±0.25%	(C)
Violet	7	7	7	10MΩ	±0.10%	(B)
Grey	8	8	8		±0.05%	
White	9	9	9			
Gold				0.1	± 5%	(J)
Silver				0.01	± 10%	(K)

0.1%, 0.25%, 0.5%, 1%

237Ω ± 1%

5-Band-Code

Measuring Resistance…

Select "DMM" mode, then "Ohms."
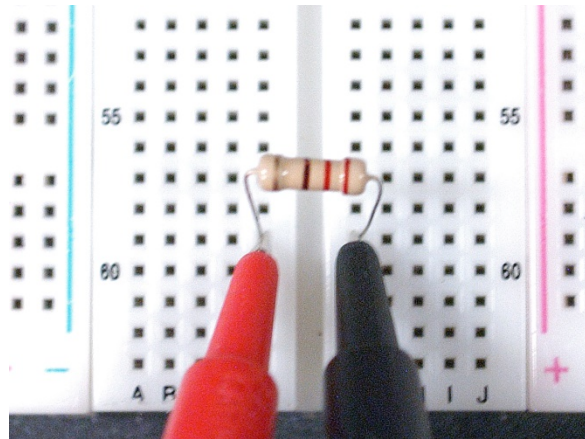
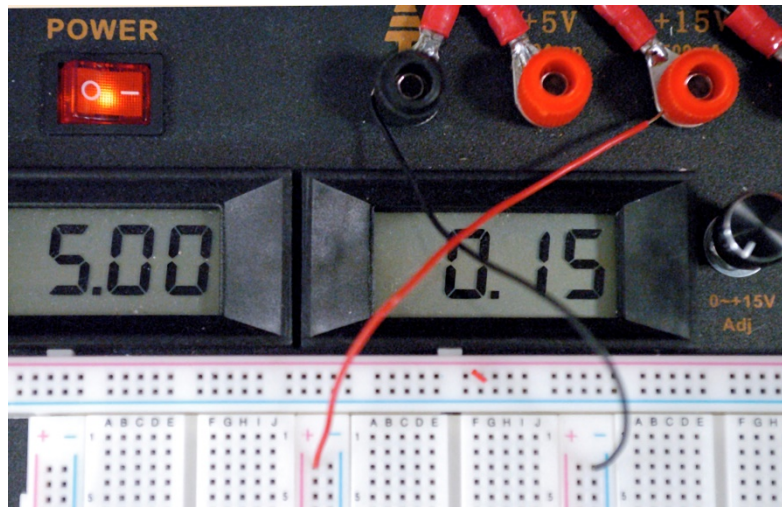Place leads across the 220 Ohm 5% Resistor on a breadboard. (Note Bands: Red-Red-Brown-Gold)

Measured resistance is ~216 ohm which is within 5% tolerance.
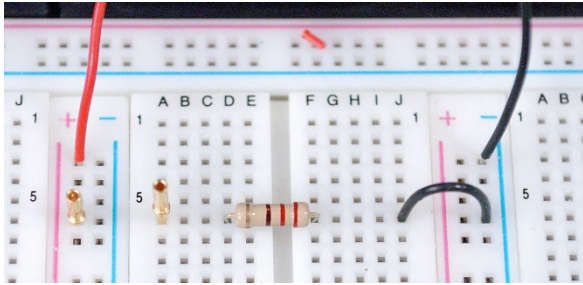
Setting up 5 VDC on the Powered Breadboard…

Set the left-most power supply to 5VDC, and jumper from the binding posts to the breadboard rails (red is +, blue is GND (-).

You can double check the voltage using the Hantek meter. Select the "DMM" and "DC V", then probe the insides of the appropriate binding post. The meter is *parallel* with the power source.

Measuring Current…

Straddle the resistor across the breadboard gap, and jumper to ground. Add test points for measuring current (the meter is in *series* with the circuit).

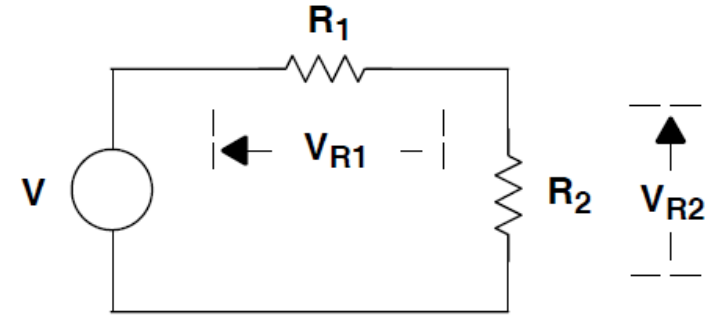To measure current, move the red lead over to the "mA" terminal. Select "DMM" and "DC (ma)". Place the test leads across the test points.
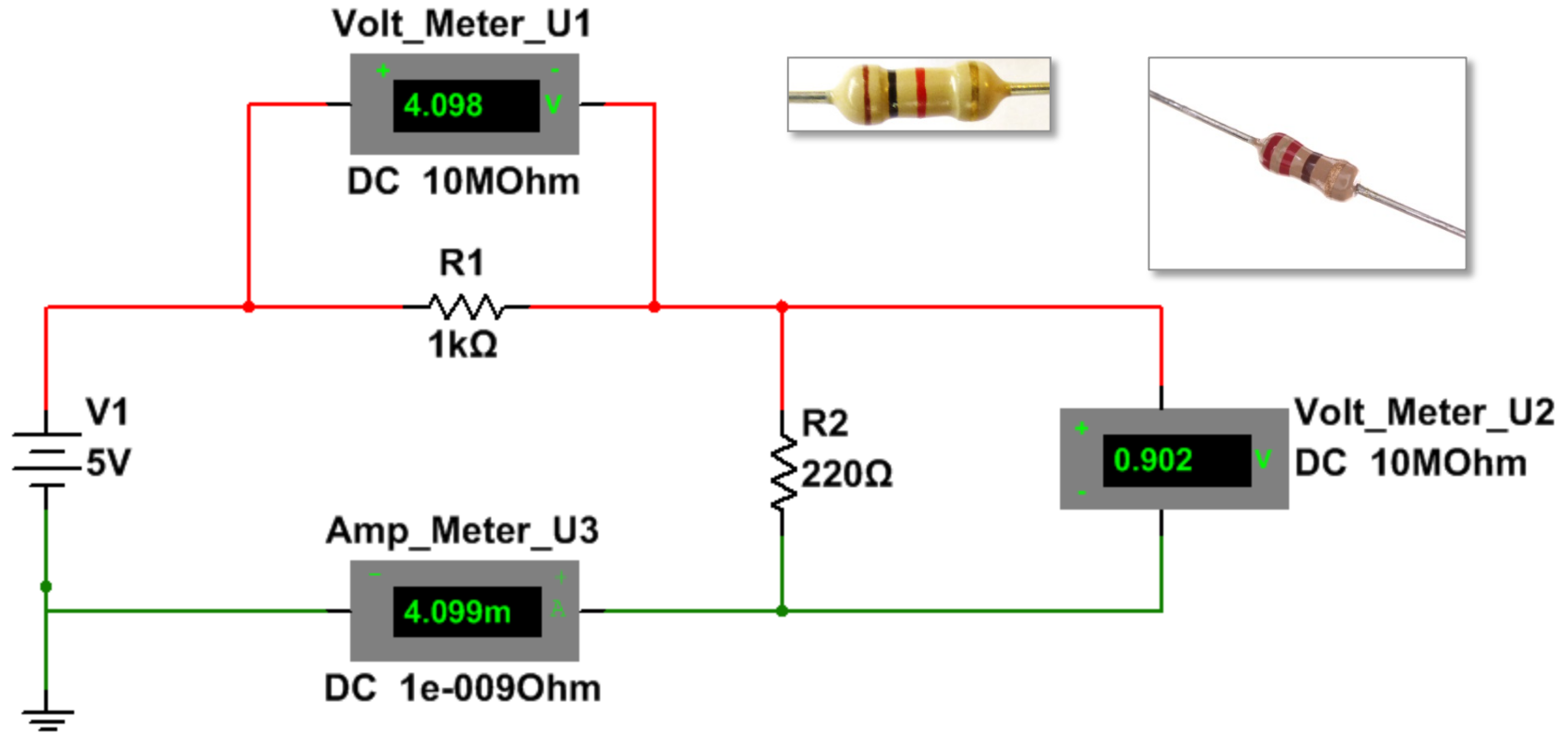

The measured current is 23 ma, as expected. When done, return the red test lead to the V/Ohm/C/Diode terminal.

Kirchoff's Voltage Law

- Kirchoff's Voltage Law:
  - $\sum V_{Source} = \sum V_{Drops}$
  - The sum of the voltage drops in a series circuit equals the sum of the voltage sources.
  - $V = V_{R_1} + V_{R_2}$
- For series resistors, $R_{Total} = R_1 + R_2 + \cdots + R_n$
- For example, if V = 5 V, R_1 = 1000 Ω, and R_2 = 220 Ω, then the voltage drop across each resistor is determined as follows:
  - Total resistance, R = 1220 Ω.
  - $\therefore I = \dfrac{V}{R} = \dfrac{5}{1220} \sim 4.1 \; mA$
  - The voltage drop across R_1 is:
    - $V_{R_1} = IR_1 = .0041 \; x \; 1000 = 4.1 \; V$
  - The voltage drop across R_2 is:
    - $V_{R_2} = IR_2 = .0041 \; x \; 220 = .9 \; V$
  - Note that 4.1 V + .9 V = 5 V

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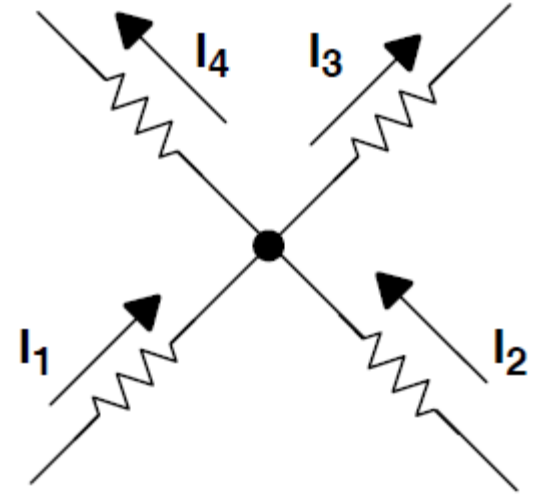
Image courtesy of Texas Instruments

Kirchoff's Voltage Law Simulation…

Kirchoff's Current Law

- Kirchoff's Current Law:
  - $\sum I_{In} = \sum I_{Out}$
  - The sum of the currents entering a junction equals the sum of the currents leaving the junction.
  - $I_1 + I_2 = I_3 + I_4$

Image courtesy of Texas Instruments

Voltage Divider Rule

- Notice that the output voltage in this circuit is simply the voltage drop across R_2.

- $V_{Out} = IR_2 = \dfrac{V}{R_T}R_2 = V\dfrac{R_2}{R_1+R_2}$

- Also notice that the output circuit will eventually provide additional pathways for the current to flow!

Image courtesy of Texas Instruments
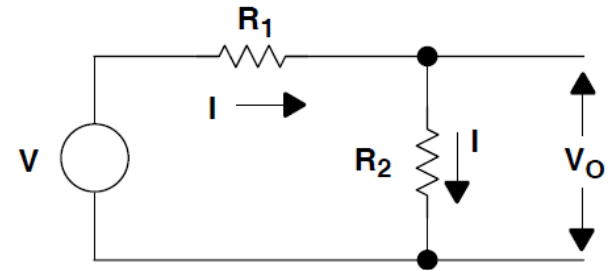
Parallel Resistors & Current Flow

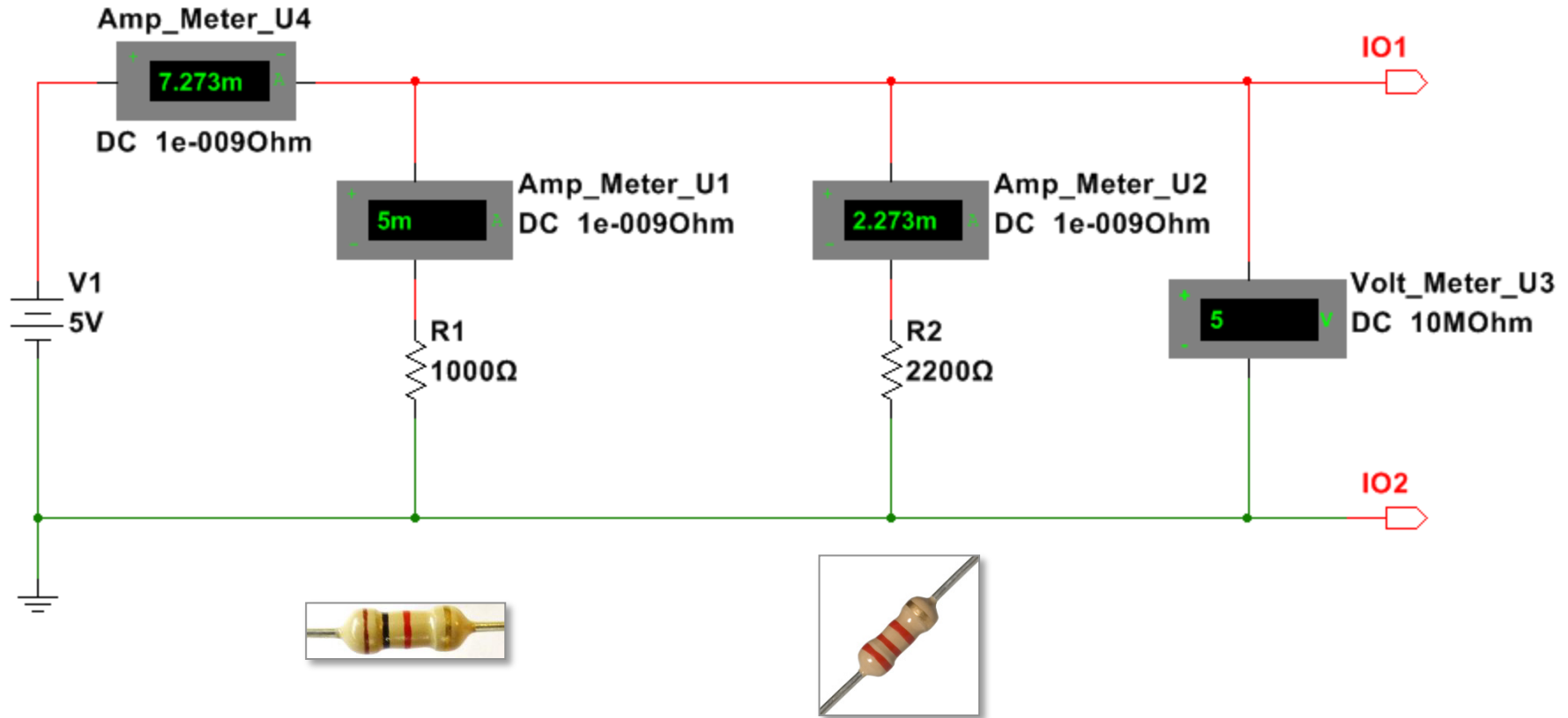- Notice that current will flow through both resistors (plus any future additional output circuitry).
- Total resistance for parallel resistors is as follows:
  - $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}...+ \frac{1}{R_n}$
  - and $I = I_1 + I_2 = \frac{V}{R}$
  - $I_1 = \frac{V}{R_1}$ and $I_2 = \frac{V}{R_2}$, also $I_1 = I\frac{R_2}{R_1+R_2}$ and $I_2 = I\frac{R_1}{R_1+R_2}$
- For example, if V=5 V, R_1=1000 Ω and R_2=2200 Ω, then:
  - $\frac{1}{R} = \frac{1}{1000} + \frac{1}{2200}, R = \frac{R_1 R_2}{R_1+R_2} \sim 688\ \Omega$
  - $I_1 = \frac{5}{1000} = 5\ mA$
  - $I_2 = \frac{5}{2200} = 2.27\ mA$
  - $\therefore I = I_1 + I_2 = 5 + 2.27 = 7.27$ mA
  - Confirm: $I = \frac{V}{R} = \frac{5}{688} \sim 7.27$mA
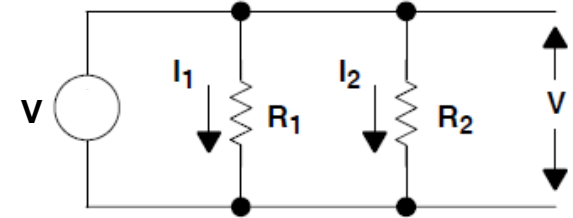
Math refresher:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1}{R_1 R_2} + \frac{R_2}{R_1 R_2} = \frac{R_1 + R_2}{R_1 R_2}$$

$$\therefore R = \frac{R_1 R_2}{R_1 + R_2}$$

Image courtesy of Texas Instruments

Parallel Resistor Current Simulation...

Series vs Parallel Resistor Simulation…

Amp_Meter_U1

0.021

DC 1e-009Ohm

R1
150Ω

V1
5V

RED LED Specs:
Vf = 1.83V @ 20mA

LED1

Volt_Meter_U2
DC 10MOhm

1.829

+ (longer lead)

-

ANODE   CATHODE
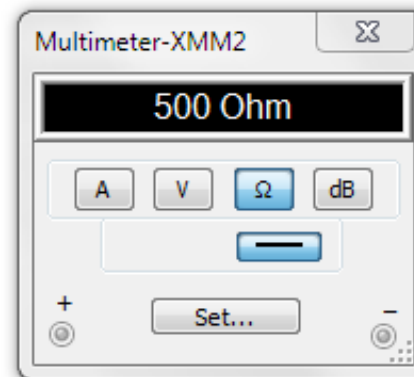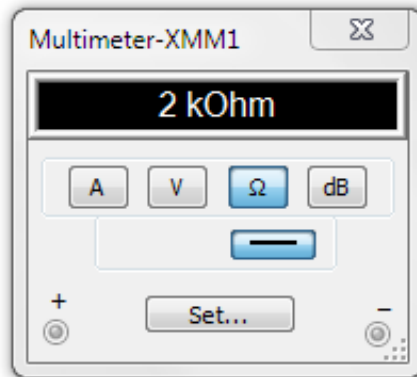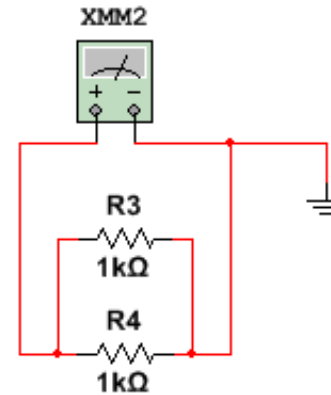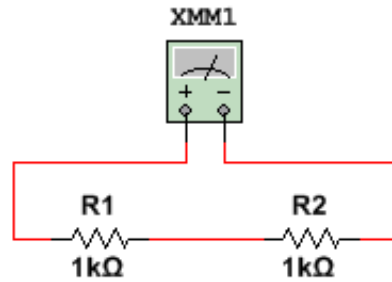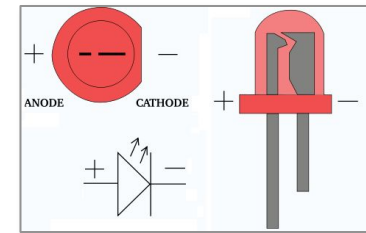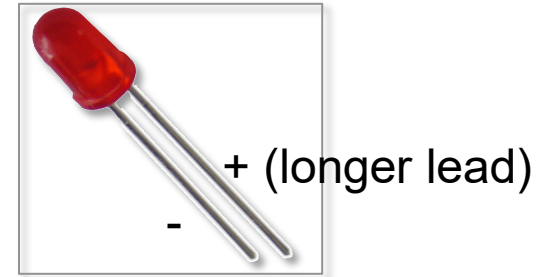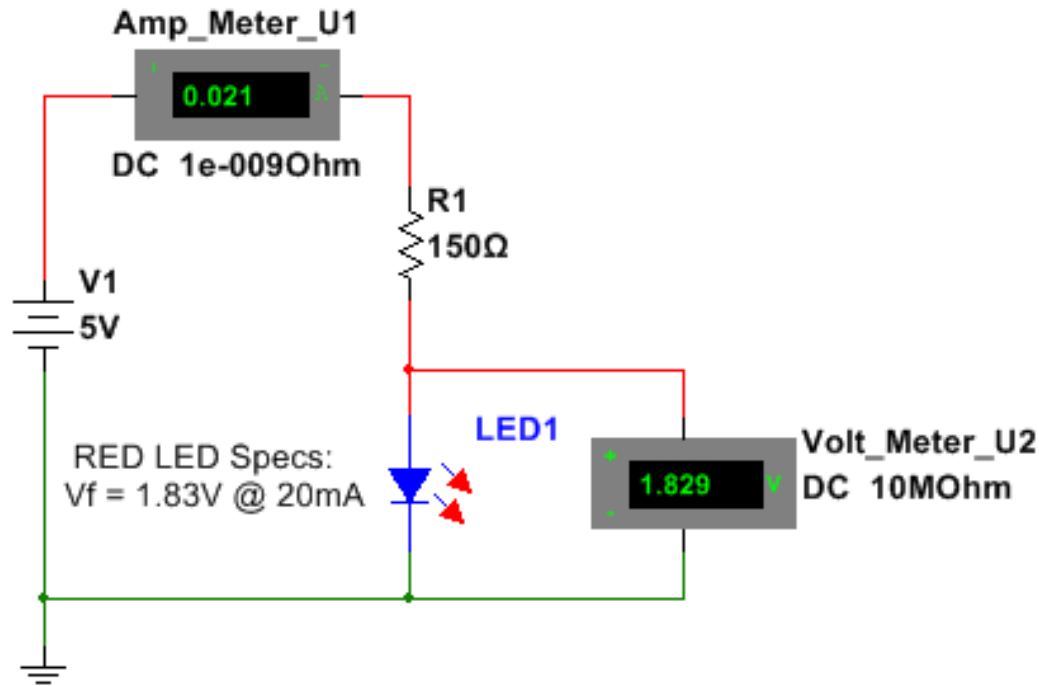
Power…

- The power through a resistor:
  - $P = I^2 R = VI$ in watts.
- Resistors must dissipate heat, and are rated based on watts – typically 1/8, ¼, ½ or a 1 watt resistor.
- The resistor may be made of carbon composition, carbon film, metal film, metal oxide film, foil or even wire wound.

Capacitance

- Capacitors are made of two parallel plates, separated by a dielectric material. These may be foil plates and rolled in a tubular shape or simply bonded as flat metal plates.
    - The charge on a capacitor is: $Q = C\ x\ V,$ where
        - Q is the charge in Coulombs,
        - C is the capcitance in Farads and
        - V is the voltage in Volts.

- $C = \varepsilon_r \varepsilon_0 \dfrac{A}{d}$, *where*
    - C is the capacitance, in farads;
    - A is the area of overlap of the two plates, in square meters;
    - ε_r is the relative static permittivity (sometimes called the dielectric constant) of the material between the plates (for a vacuum, ε_r = 1);
    - ε_0 is the electric constant $\varepsilon_0 \approx 8.854\times10^{-12}$ F/m; - farads per meter.
    - d is the separation between the plates, in meters;

Image courtesy of Eric Schrader, Wikipedia

RC Charging of a Capacitor...

- Note the exponential rise in voltage across the capacitor as it charges.
- The oscilloscope is set at 1V/div and 50ms/div.
- The voltage across the resistor and current flow both fall as the capacitor charges.
- $V(t) = V_0(1 - e^{-\frac{t}{RC}})$

  $Q = CV_0(1 - e^{-\frac{t}{RC}})$

  $I = \frac{V_0}{R} e^{-\frac{t}{RC}}$

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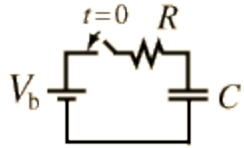
Charging…

$V_b = V_R + V_C$

$V_b = IR + \dfrac{Q}{C}$

As charging progresses,

$V_b = IR + \dfrac{Q}{C}$ ⬆

⬇

current decreases and charge increases.

$Q = CV_b\left[1 - e^{-t/RC}\right]$
Charge on capacitor

$I = \dfrac{V_b}{R} e^{-t/RC}$
Charging current
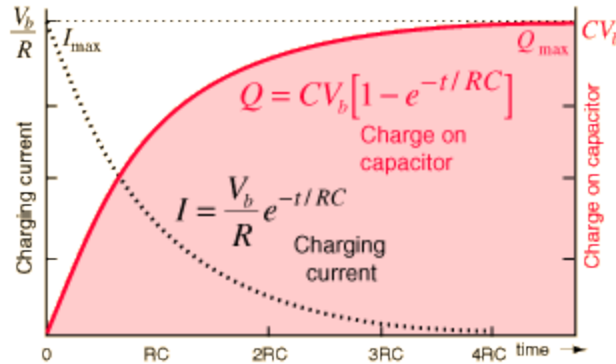
At $t = 0$
$Q = 0$
$V_C = 0$
$I = \dfrac{V_b}{R}$

As $t \to \infty$
$Q \to CV_b$
$V_C \to V_b$
$I \to 0$

From previous example:
R=750 Ω
V_0=5 Vdc
C=100 µf
RC=0.075s
    (i.e. t=.2s is 2.7 "RC"s
I_{max}=0.00667 amps
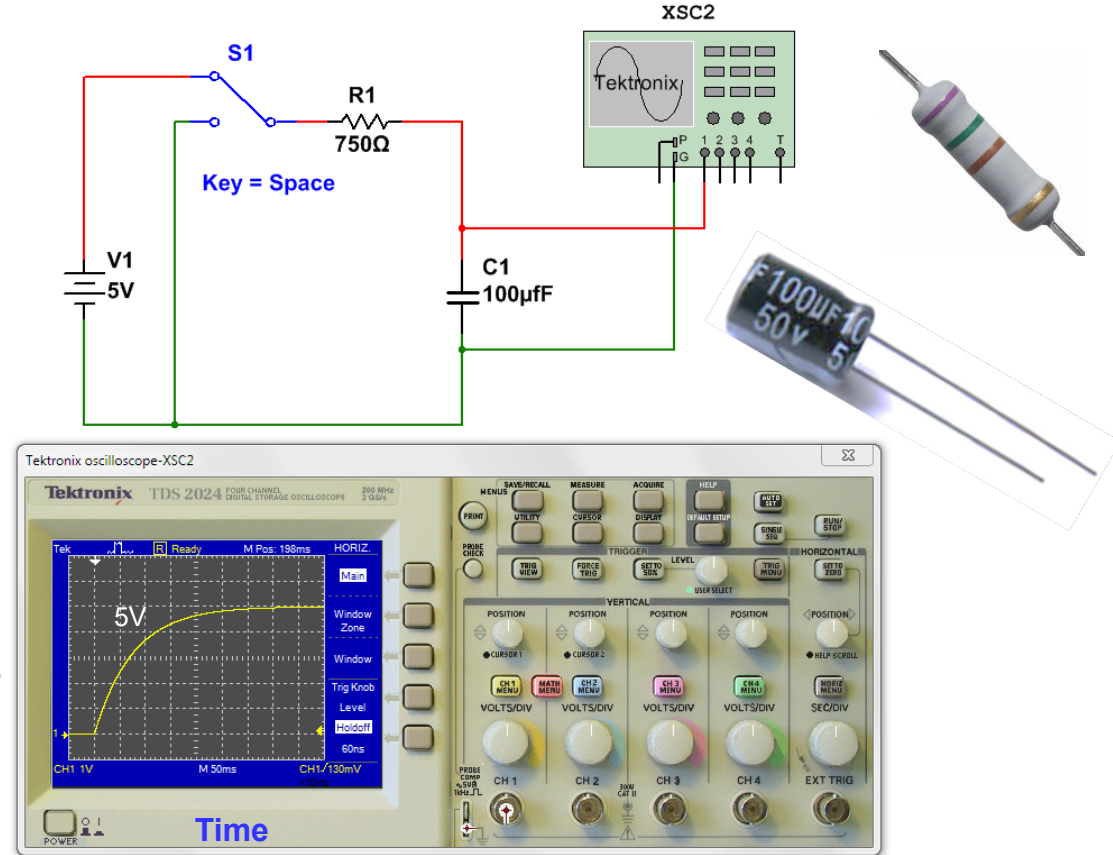   (Upon closing switch.)
Q_{max}=500 µC

RC Discharging of a Capacitor…

- Note the exponential fall in voltage across the capacitor as it discharges.
- The oscilloscope is set at 1V/div and 50ms/div.
- At the onset of discharge, voltage across the resistor is the capacitor voltage, and then rapidly decreases along with the current.

- $V(t) = V_0 e^{-\frac{t}{RC}}$

  $Q = CV_0 e^{-\frac{t}{RC}}$

  $I = \frac{V_0}{R} e^{-\frac{t}{RC}}$

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Discharging…

$$V_C = V_0, \quad Q = CV_0, \quad I = \frac{V_0}{R}$$

$t=0$

V_0, C, R

$$V_C = \frac{Q}{C} = IR$$

The voltage V_C, the current I, and the charge Q all follow the same type of decay curve when the switch is closed.

$$V_C = V_0\, e^{-t/RC}$$

$$Q = CV_0\, e^{-t/RC}$$

$$I = \frac{V_0}{R}\, e^{-t/RC}$$

0   RC   2RC   3RC   time
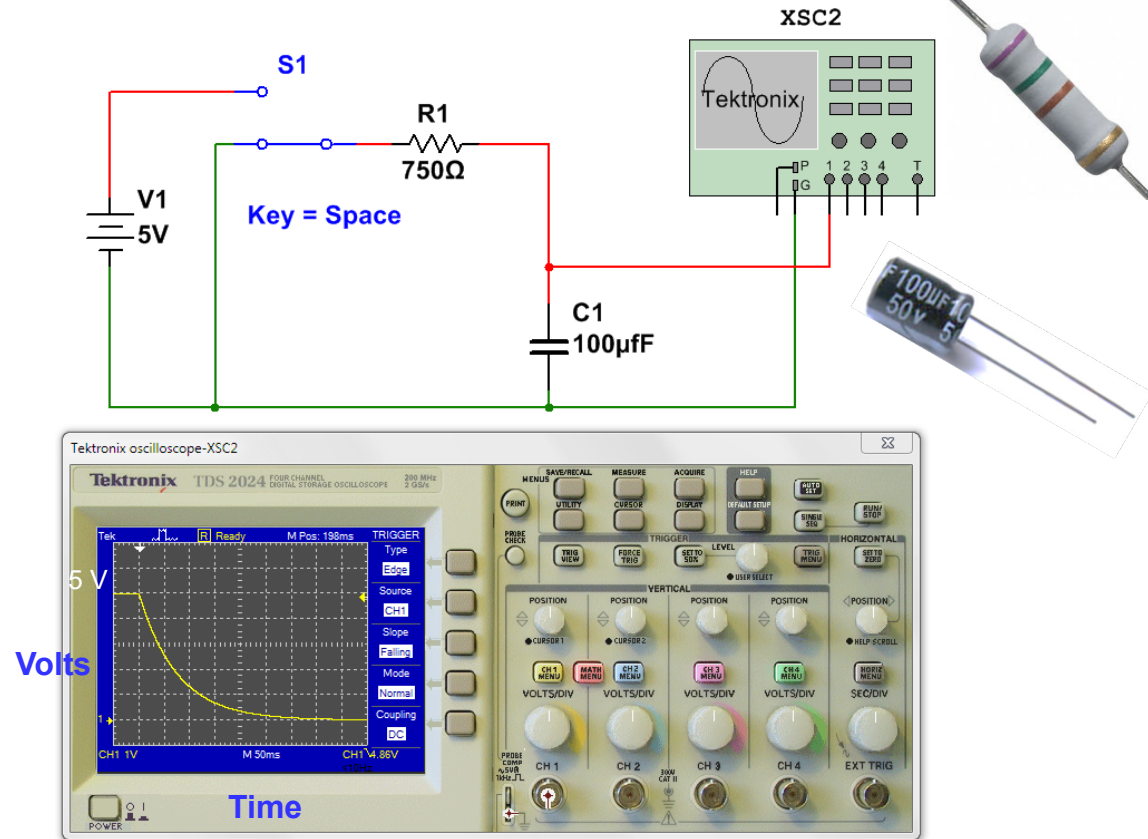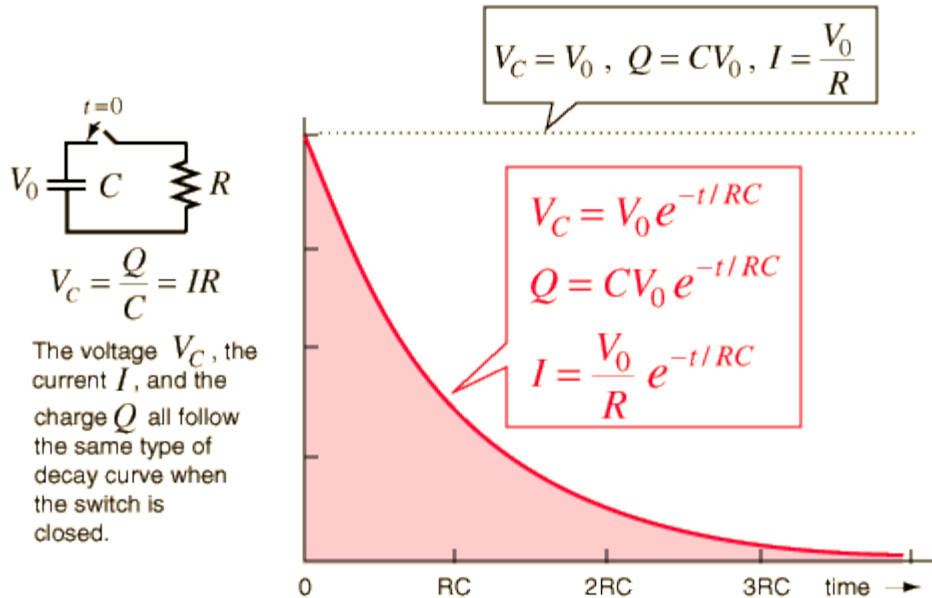
From the previous example:
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C=100µf
RC=0.075s
    (i.e. t=.2s is 2.67 "RC"s
I_{max}=0.00667 amps
    (Upon closing switch.)
Q_{max}=500µC (the starting charge)

Parallel vs Series Capacitors

- To calculate the equivalent capacitance of capacitors in *parallel*, use the following formula: $C_{Total} = C_1 + C_2 + \cdots + C_n$

- For *series* capacitors use:

$$\frac{1}{C_{Total}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}$$

- Notice the similarity to resistors in *series* and *parallel* respectively (just reversed).

Summary

- Circuit Theory (Using National Instruments Multisim* Software).
  - Kirchoff's Voltage and Current Laws
  - Voltage Divider Rule
  - Resistances - Series and Parallel Resistors
  - Capacitance
    - Series and Parallel Capacitors
    - Charging and Discharging a Capacitor
  - *Self Review - Advanced Topics*
    - Inductors
    - Impedance (Z) and Admittance (Y)
    - Thevenin's Theorem
    - Superposition Theorem

Advanced Topic: Inductors or *Chokes*

- Current flowing through a wire-wound coil produces a magnetic flux.
  - A secondary voltage is induced into the coil by the movement of the magnetic flux as it opposes or resists any changes in the electric current flowing through it.
  - An inductor opposes the rate of change of current flowing through it.
- Inductance is the amount of voltage dropped across an inductor for a given rate of change of current flowing through it.
  - The unit of inductance is the Henry (H).

Determinants of the Self-Induced Voltage…

$$V_L = N\frac{d\Phi}{dt} = \frac{\mu N^2 A}{l}\frac{di}{dt}$$

Where:

N is the number of turns,

A is the cross section in m^2,

Φ (phi) is the amount of flux in webers,

μ (mu) is the permeability of the core material

l is the length of the coil in meters and

$\frac{di}{dt}$ is the rate of change of current in A/S

Air-cored fixed Air-cored preset Air-cored variable Iron-cored fixed Iron-cored transformer

Ferrite-cored fixed Ferrite-cored preset Ferrite-cored variable Ferrite-cored transformer

Images courtesy of talkingelectronics. com and matrixtsl.com

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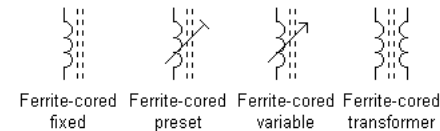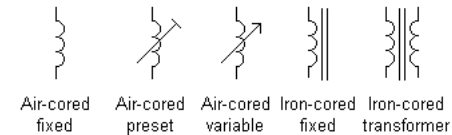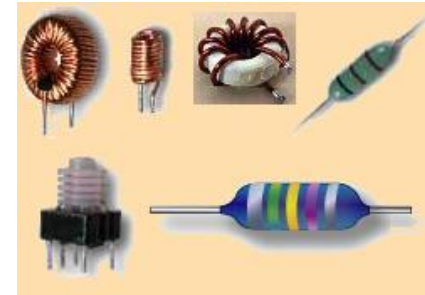
Calculating the Back-EMF…

$$V_L(t) = \frac{d\phi}{dt} = \frac{dLi}{dt} = -L\frac{di}{dt} = \text{inductance times rate of current change}$$

Where:
L is the self inductance and
$\frac{di}{dt}$ *the rate of change of current*

- A circuit with an inductance of one H and current flow changing at a rate of one amp per second, will have an emf of one volt.
- With a dc current, $\frac{di}{dt} = 0$, and the induced emf voltage will be zero.
  - With a steady dc current flowing, the inductor acts as a wire (short-circuit)
- Other concepts for independent study include power and energy storage in the magnetic field.

Series Inductors – No Mutual Conductance…

- Consider series inductors in which there is *no mutual inductance* or *magnetic coupling* between the individual inductors.
- Inductors will have a common current flowing.
- The total inductance is the sum of the individual inductors:
$L_T = L_1 + L_2 + \cdots + L_n$
  - For example, if L_1=20mH and L_2=30mH, then L_T= 50mH.
  - This is derived by calculating the sum of the individual voltage drops and dividing by $\frac{di}{dt}$:
    - $L_T \frac{di}{dt} = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + \cdots + L_n \frac{di}{dt}$

Images courtesy of AspenCore, Electronics Tutorials

Mutual Coupling…

Cumulatively Coupled Series Inductors
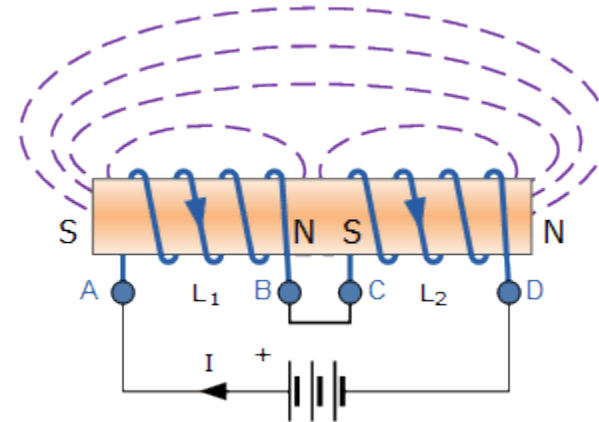
Differentially Coupled Series Inductors

Prof. Steven S. Saliterman

Images courtesy of AspenCore, Electronics Tutorials
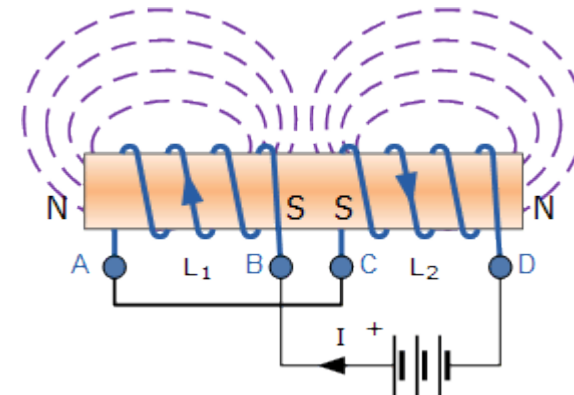
Allowing for Mutual Inductance…

- In the real world you need to allow for mutual inductance.
    - This may increase or decrease the total inductance depending on distance apart and orientation.
    - For cumulatively coupled coils*:

$$L_T \frac{di}{dt} = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + 2M \frac{di}{dt} \text{ or}$$

$$L_T = L_1 + L_2 + 2M$$

(*minus 2M for differentially coupled inductors)

Example…

- Consider two inductors in series, with inductance of 20mH and 60mH, and with measured total inductance of 100mH. What is the mutual inductance (M)?

  - $L_T = L_1 + L_2 + 2M$
  - $100 = 20 + 60 + 2M,$ therefore

  $M = \dfrac{20}{2} = 10mH$

Parallel Inductors…

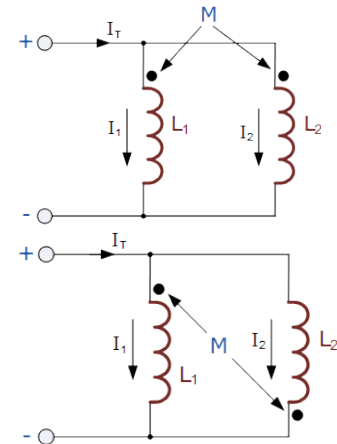- For parallel inductors, the total inductance can be calculated as follows:

$$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_n} \text{ or}$$

$$L_T = \frac{L_1 L_2}{L_1 + L_2} \text{ (if two inductors only)}$$

- Allowing for mutual inductance:

$$L_t = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} \text{ (mutually aiding)}$$

$$L_t = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M} \text{ (mutually opposing)}$$

Images courtesy of AspenCore, Electronics Tutorials

Example…

- Consider two parallel inductors with values of 50mH and 25mH respectively, and mutually aiding. The mutual inductance is given as 11mH.

  - $L_T = \dfrac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} = \dfrac{50mH \; x \; 25mH - 11mH^2}{50mH + 25mH - 2x11mH} = 21.3mH$

Impedance

- Impedance (Z) – Total opposition a device or circuit offers to flow of an *alternating current* at a given *frequency*.
  - Units are in ohms
  - Complex quantity graphically shown on a vector plane (real and imaginary parts): $R + jX$ where R is *resistance* and X *reactance*

Admittance…

- Admittance (Y) – the reciprocal of impedance.
  - Units are in siemens (S)
  - Complex quantity (real and imaginary parts):

Superposition Theorem

$$Y = \frac{1}{Z} = G + jB$$

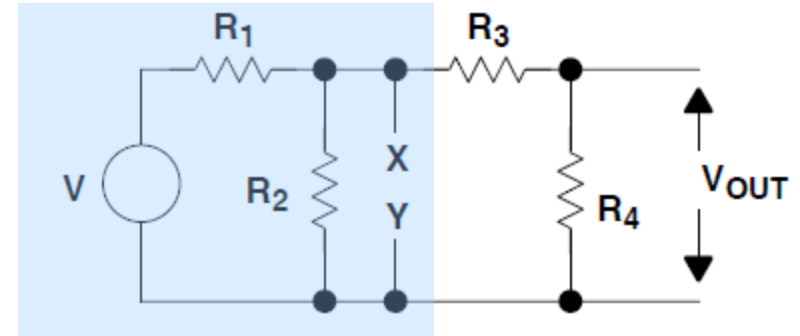*where G is conductance and B susceptance*

Thevenin's Theorem

- A means for isolating a portion of a circuit to simplify analysis, rather than solving loop and node equations.

- Used when the input source is a voltage (in contrast to *Norton's theorem* which is used when the source is a current).
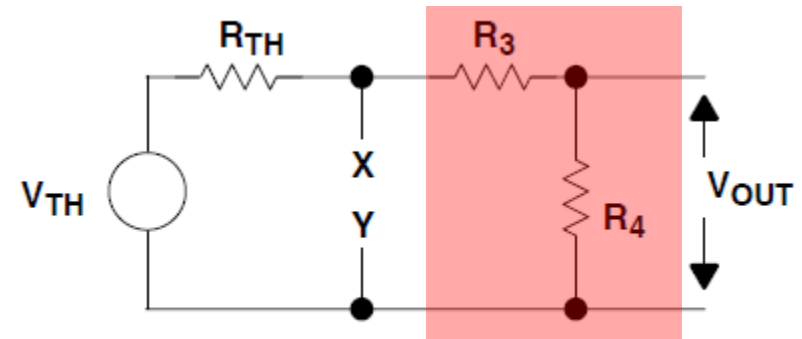
Applying Thevenin's Theorem...

- How do we calculate the output voltage?
- We can isolate the area in blue and consolidate it to a simple series circuit with a resistance, or *impedance* equivalent.
- Start by calculating the output voltage at X and Y, considering only the blue shaded area. This is simply the voltage drop across R_2, or $V_{TH} = V \frac{R_2}{R_1+R_2}$ (the voltage divider rule).
- Now *short* the voltage source and calculate the equivalent resistance, or $R_{TH} = \frac{R_1 R_2}{R_1+R_2}$ (our equation for parallel resistors).
- Finally, considering the circuit in red, applying the voltage divider rule again, we can calculate the output voltage:

$$V_{Out} = V_{TH} \frac{R_4}{R_{TH}+R_3+R_4}$$

(a) The Original Circuit

(b) The Thevenin Equivalent Circuit

Prof. Steven S. Saliterman   Mancini, R . *Op Amps for Everyone*, Texas Instruments, Dallas, TX (2002)

Superposition Theorem

- How do we calculate an output voltage when there are more than one voltage source?
  - When there are independent sources, the voltages and currents resulting from each source can be calculated separately, and the results are added algebraically.
  - Simply image all but one source as being shorted to ground, calculate the output and current and repeat for each source. The output voltage is then the sum of the individual outputs.