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Particle migration in nanofluids: A critical review

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ABSTRACT

Particle migration in nanofluids has received less than due attention in the literature, and is generally an open research topic requiring more investigation. Particle migration can have great influences on the characteristics of nanofluids through disturbing the distributions of nanoparticle concentration and thermophysical properties. This paper attempts to review and summarize the studies conducted on nanofluids, considering particle migration, including those conducted via methods such as Eulerian-Lagrangian, Buongiorno model, molecular dynamics simulation, and different theoretical approaches. Several important issues are highlighted that deserve greater attention. It is shown that there are still several hot debates for flow and thermal mechanisms in nanofluids, particularly regarding the behavior of nanoparticles. Besides, this survey identifies the challenges and opportunities for future research.

1. Introduction

Nanofluids have emerged as an interesting and novel class of nanotechnology-based heat transfer fluids and have grown significantly in the past few years. Compared to conventional fluids, the superior thermal conductivity and better convective heat transfer as well as little pressure drop have made nanofluids one of the most promising emerging technologies in heat transfer applications. Therefore, there is much excitement about applying nanofluids to meet new challenges in cooling techniques and thermal management of high heat flux equipment. There is, however, only limited knowledge of the mechanisms by which these improvements are evaluated, and how several features (such as particle clustering, interactions with the walls, particle migration, and so forth) affect the behavior of nanofluids. In spite of the substantial amount of effort invested in this area, a satisfactory theoretical explanation has not yet been fully provided for possible heat transfer enhancement mechanism related to nanofluids. Researchers are being challenged to discover the many unexpected hydrothermal characteristics of these fluids, to suggest new mechanisms and unconventional models to explain their behavior. The broad differences in the results presented by different researchers for the same type of, and similar, nanofluids increase the complexity of the problem further.

The number of publications on this subject in the recent years has increased so incrementally such that the number of publications has increased from a few ones before the year 2000 to several hundred in recent years. In spite of fifteen years of serious research effort, the vision of the widespread applicability of nanofluids is still a promise rather than a reality. Despite the ever increasing nanofluid research projects, the state of nanofluid research has not yet clearly been determined, and even has proved to be confusing in several cases. Many inconsistent experimental results existing alongside unproven hypothetical theories in the relevant literature is indicative of the above statement. There are many challenges that need to be pursued in the future. Some of the studies conducted on nanofluids have yielded incomplete results, some have delivered conflicting results, and some have produced doubtful results.

The state of the art in the research of nanofluids is still in its initial phases. A review carried out by Ozerinc et al. [1] shows significant inconsistencies among the available experimental data, and between the experimental results and the predictions of theoretical models. Many reasons may account for this situation. The complex nature of nanoparticles, even the more complex nanoparticle-based fluid interactions as well as different preparation methods employed in experimental investigations, that are often coupled with diverse surfactants, are among reasons resulting in such inconsistencies in research projects on nanofluids. Another significant reason can be the fact that nanofluids are commonly considered by many researchers as a homogenous (single-phase) medium with a uniform distribution of

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nanoparticles. Such an approach can lead to great errors in nanofluid simulation results. In fact, one of the main factors accounting for inconsistencies prevalent in nanofluid research projects is ignoring the effects of particle migration on flow and heat transfer characteristics.

Flow-induced particle migration has been attributed by some researchers as a probable important mechanism for enhanced heat transfer in nanofluids. Particle migration modifies profiles of velocity and thermophysical properties by disturbing the particle concentration distribution; this, as a result, can change the heat transfer rate. Physically, the particle migration idea is to consider that nanofluids are a heterogeneous two-phase mixture. Its success in predicting laminar flow and heat transfer reveals that particle-fluid interaction missing in the widely applied homogeneous models of nanofluids should be explored further to show the essential mechanisms of thermal transport in nanofluids. In order to improve our understanding of convection heat transfer of nanofluids, it is critical to understand the dynamics of nanoparticles.

Particle migration in conventional suspensions (i.e., suspensions containing millimeter and micrometer particles) has been subject to many studies. The studies show that suspensions with spherical particles have a non-uniform concentration distribution in a nonhomogeneous shear flow [2]. In suspensions with rather larger particles, factors such as non-uniform shear rate and viscosity gradient affect particle migration. However, as particles get smaller, factors such as Brownian motion and thermophoresis, in addition to above mentioned parameters, gain some significance.

Unlike suspensions containing micron-sized particles, a few studies have evaluated the particle migration in nanofluids. There are very few studies conducted on suspensions of particles for which Brownian motion is important. For Brownian suspensions, Brownian motion can have a significant effect on the cross-stream migration, as concluded by Frank et al. [3]. Wen and Ding [4] examined the movement of nanoparticles in laminar pressure-driven pipe flows considering Brownian motion for dilute suspensions. They demonstrated that the particle concentration near the wall is noticeably smaller than that at the tube center.

With changes in particle size, even the dependency of particle migration on flow rate undergoes some changes. For instance, Semwogerere et al. [5] presented the concentration profile of suspensions of Brownian particles. They demonstrated that, in contrast with non-Brownian suspensions, flow rate has a significant effect on the concentration profile in a Brownian suspension. Regarding the important effect of particle size, the results of previous studies (i.e., the studies performed on suspensions containing particles larger than nano-scale) cannot be applied to suspensions containing nanoparticles (i.e. nanofluids).

It is very important to understand the flow behavior and particle migration in nanofluids in order to make application of nanofluids feasible in the near future. Particle migration can affect overall heat transfer performance. When concentration shows non-uniform distribution, the effective properties will incorporate non-uniform distributions. The convective heat transfer coefficient and pressure drop are significantly affected by the values of near-wall thermal conductivity and viscosity. Therefore, it is crucially important to consider particle migration in the analysis of nanofluid problems, which has thus far been overlooked in most of the studies carried out on nanofluids.

Most of the studies have considered a uniform particle distribution for nanofluids [6,7]. However, particles can migrate under conditions of shear and viscosity gradient [8], such that using uniform concentration will introduce errors to the results. Nanoparticle motion (believed to have a key contribution to enhanced heat transfer) is governed by superposition of several effects (thermophoresis, Saffman lift force, Brownian motion, Soret and Dufour effects, and so forth), some of which are not yet fully understood since they only become important at very small length scales. Studying particle migration in nanofluids can help better understand the physics concepts behind nanofluids, and also decrease some of the inconsistencies existing in the current literature.

It is not the objective of this survey to generally review the related literature of nanofluids, since it has been done in a number of recent publications [9–12]. In fact, this paper aims to review the conducted studies in which particle migration in nanofluids has been taken into consideration. Particle migration is a subject which is less studied in related investigations conducted on nanofluids. This review paper also identifies the existing challenges and opportunities in this area and attempts to present directions for future studies. The author of this paper hopes that the current contribution can provide a brighter path for further research on nanofluids.

2. Particle migration in conventional suspensions

Suspensions have motivated a great number of researchers to study this subject due to their widespread applications in industry. These studies are focused on suspensions containing particles of micron and millimeter sizes, in which Brownian motion does not usually play a key role. Before evaluating the subject of particle migration in nanofluids, the most significant studies conducted on conventional suspensions considering particle migration are introduced.

Particle migration is an important issue in suspensions in a vast range of engineering applications such as composite materials, transport of sediments, heat transfer, oil recovery, chromatography, sequestration processes in porous media, and even flow of blood. Research studies on the cross-stream migration of particles have occupied a significant position in suspension rheology since the study performed by Segre and Silberberg [13] in 1962 on the inertial migration of particles in a tube flow, and the study conducted by Leighton and Acrivos [8] in 1987 on the particle–particle interaction in a concentrated suspension.

Several studies have been conducted on the mechanisms of particle migration in suspensions containing micron-sized particles both by experiment [14,15] and combined simulation and modeling [16,17]. Many models have been suggested for the study of suspension flows. The continuum models that are applied to explain the particle migration fall basically into two categories: the diffusive flux model by Phillips et al. [18], and the suspension balance model by Nott and Brady [19]. These models have attained some level of success in predicting qualitative features of migration process and concentration distribution.

Phillips et al. [18] applied the scaling arguments of Leighton and Acrivos [8] to develop the so-called diffusive flux model. In this model, particle migration results from gradients in viscosity, shear rate, and concentration. This model was modified by Fang et al. [20] to account for the various rates of migration in the shear plane. In an alternative modeling method based on the conservation laws, designated as the balance model of suspension, the stress in particle phase is expressed via a constitutive equation, and particle transport is evaluated by rheological models. The suspension balance model was refined by Morris and Boulay [21] and Shapley et al. [22] to explain non-isotropic migration rates and to progress the modeling of particle velocity fluctuations. Applying this model and the finite volume method, Miller and Morris [23] simulated the pressure driven flow of a non-colloidal suspension in a two-dimensional geometry and axisymmetric circular channels. Furthermore, Miller et al. [24] modified the established shear-based rheological model of Morris and Boulay [21] and introduced a

frame-invariant formulation for the suspension balance model in general geometries. Mirbod [25] utilized the suspension balance model to present numerical validation of the particle migration in a concentrated suspension between rotating eccentric cylinders. The simulation method was validated using an available analytical solution in a circular Couette flow. The simulation results of the concentration distribution and the velocity profile showed a proper agreement with the experimental data. As is observed in Fig. 1, the inner cylinder was rotating with angular velocity ω. Fig. 2 illustrates concentration profiles in the eccentric cylinders as a function of turns of inner rod for average concentration of 50%. It can be seen that as the time increases, a large region of concentrated particle phase forms in the wide-gap and leads to a general migration of particles away from the inner rod towards the outer cylinder.

Other rheological models based on mixture theory have also been suggested by Buyevich [26] and Pozarnik and Skerget [27].

The shear- and viscosity-induced effects disturb the suspension from its equilibrium state. The pressure-driven flow of concentrated suspensions within a channel tends to shear-induced particle migration. When suspensions are pumped within a pipe, it has been noticed that the concentration of particles is greater in the central region and lower in the wall region [28]. For instance, the particles migrate toward the centerline in pipe flows [29]. This phenomenon is attributed to particle dispersion due to shear [8]. Particles in the high shear region tend to migrate toward the low shear region. Phillips et al. [18] suggested that particle diffusion-dispersion fluxes are comprised of three components, namely, dispersion due to shear-induced particle-particle collision, particle dispersion due to viscosity gradient in a shear field, and self-diffusion due to Brownian motion. This can lead to modifications in velocity profile and pressure drop. In addition, migration of particles to the central region may expedite the formation of clusters due to high concentration, and these structures are less likely to break as a result of low shear rates in that region.

Although constitutive equations for the modeling of dense suspensions in nonlinear shear flows have been capable of determining the correct steady-state concentration profile, none have been able to follow the transient experimentally measured concentration distribution precisely over a range of suspended particle radii with a consistent set of diffusion coefficients. Ingber et al. [30] applied two improvements to the diffusive flux model, namely, modeling the diffusion coefficients as linear functions of the so-called nonlinearity parameter and adding slip boundary conditions at the wall. With these two improvements, it was shown that the modified model can accurately predict the transient concentration in a Couette device over a wide range of particle radii.

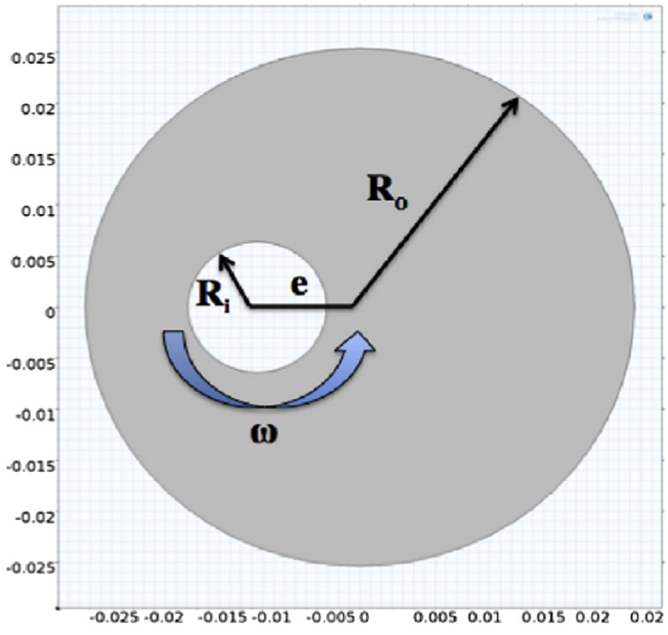

Fig. 1. Schematic diagram of the eccentric cylinders [25]. Reprinted with permission from Elsevier.

Yadav et al. [2] reported numerical simulation of shear-induced particle migration for low Reynolds number transport of concentrated suspension through Y-shaped three-dimensional bifurcation channel. It was noticed that the velocity profile for the case of concentrated suspension differs significantly from that of a Newtonian fluid (see Fig. 3). The locations of the peak velocity and concentration in the inlet and side branch were found to be significantly influenced by the bulk particle concentration and angle of bifurcation. Wall shear stress level was the highest near the bifurcation region.

Ahmed and Singh [31] provided numerical validation of particle migration during flow of concentrated suspension in asymmetric T-junction bifurcation channel observed in the experimental study of Xi and Shapley [32]. They carried out the simulation in a 3D bifurcation geometry using the diffusive flux model. It was observed that the inhomogeneous concentration distribution persists throughout the inlet and downstream channels in spanwise direction. Due to the particle migration near the bifurcation section there is almost equal partitioning of flow in the two downstream branches.

Besides analytical and numerical studies, some researchers have investigated particle migration experimentally. Othman et al. [14] measured normalized particle concentration in a microchannel for high initial particle concentrations and small particle diameters by a multi-capacitance sensing method. It was observed that the particles tend to concentrate on the center region in the case of high concentration and the large particle size.

Murisic et al. [33] carried out a systematic experimental study of settling regimes over a range of particle sizes and liquid viscosities. They focused on experiments with particle-laden thin film flows down an incline, where the effects of the viscosity of the suspending liquid and the particle size were examined. Their experimental results indicated that the particle size is a significant parameter, and the likelihood of observing the well-mixed regime increases with the decrease in the particle diameter.

Boyer et al. [34] measured experimentally normal-stress differences in dense suspensions of neutrally-buoyant spheres dispersed in a Newtonian fluid. These normal-stress differences were seen to vanish below a volume fraction of approximately twenty percent and to increase with concentration above. During the experiments, a new time-dependent behavior was noticed. This time evolution was concluded to be related to particle migration from regions of high shear to low shear. Eventually, the results were compared to the predictions of the suspension balance model.

As determined by previous studies, in addition to numerical investigations, due to the large size of particles in suspensions containing micro- and millimetric particles, particle migration has been also studied experimentally. The overall conclusion of these studies indicates the great significance of factors such as non-uniform shear rate and viscosity gradient, as well as the minor effect of parameters such as Brownian diffusion and thermophoresis diffusion on particle migration in these suspensions. Additionally, most of the studies have been performed on dense suspensions (i.e., suspensions with great concentration of solid particles), while nanofluids are known to be prepared in low concentrations. Also, particles are at a nanometric scale in nanofluids; thus, using the results related to conventional suspensions to nanofluids is quite

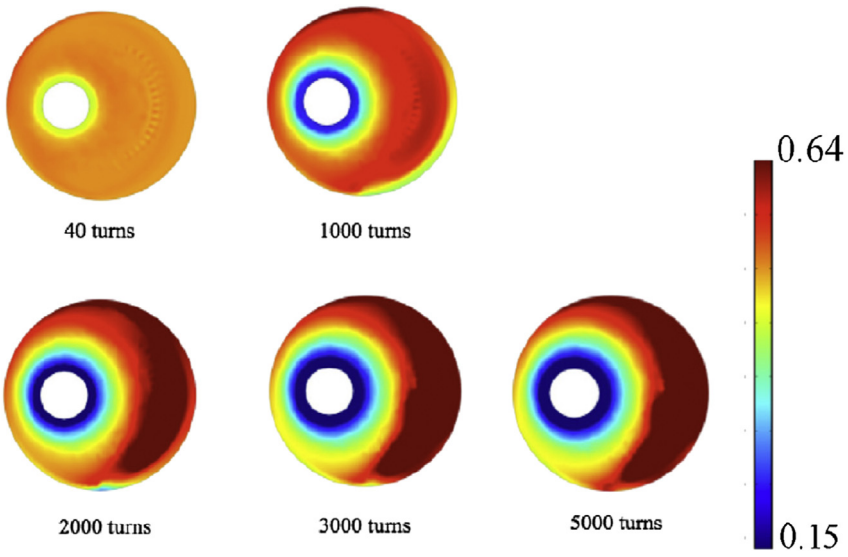

Fig. 2. Concentration profiles in the eccentric cylinders as a function of turns of inner rod for average concentration of 50% [25]. Reprinted with permission from Elsevier.

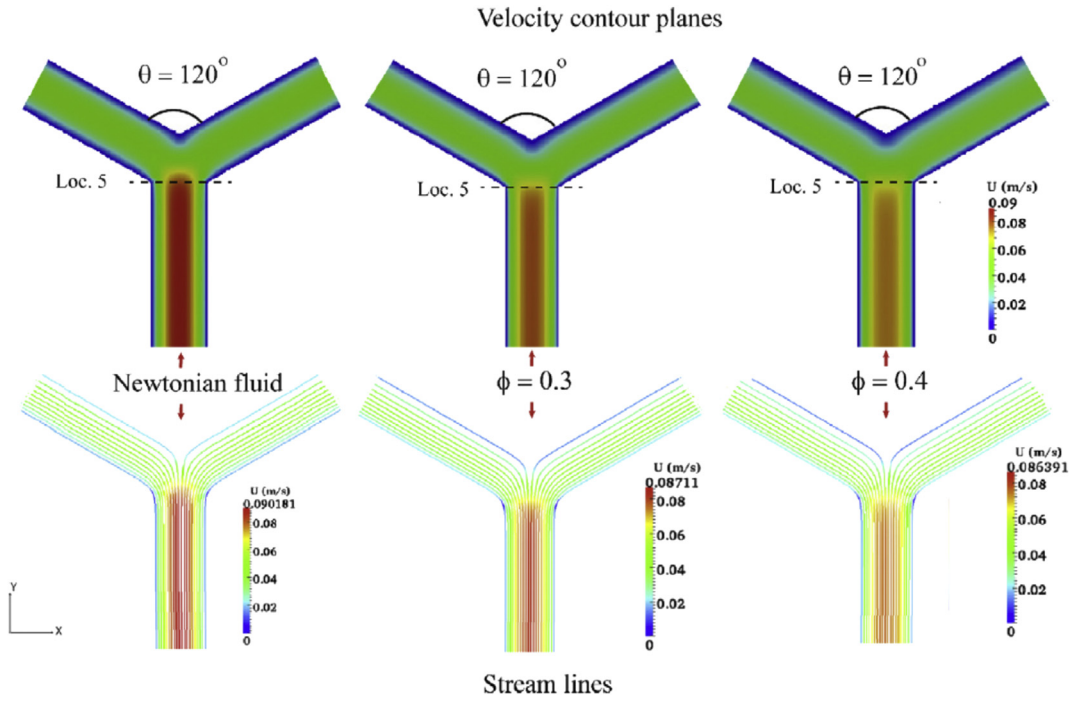

Fig. 3. Velocity contours and streamlines for Newtonian fluid and suspensions with concentrations of 30% and 40% [2]. Reprinted with permission from Elsevier.

doubtful.

3. Importance of studying particle migration in nanofluids

In the studies conducted thus far, a nanofluid was often considered to be a homogeneous fluid, and its properties were assumed to be constant in all positions of the system. These assumptions are not realistic, and may cause misunderstandings in the phenomena related to nanofluids. Even if nanoparticles are in a stationary system, they can have a Brownian motion owing to their small mass and size. Therefore, examination of nanoparticle motion is critical for evaluating nanofluids as heat transfer mediums. In addition, the results reported about hydrothermal characteristics of nanofluids are immensely contradictory. For instance, Xuan and Li [35] studied forced convection of nanofluids and reported enhancement of heat transfer, whereas Pak and Cho [36] found a somewhat different conclusion. Furthermore, Das et al. [37] claimed that nanofluids produce a lower heat transfer than base liquids. These contradicting results that are observed in the literature show that the behaviors of nanoparticles and flow may have important effects on heat transfer rate, which are presently less understood.

Although many possible reasons such as Brownian motion, ballistic conduction of phonon, liquid layering, and so forth have been proposed, there is no general mechanism to clarify the thermal behavior of nanofluids. Considering particle migration is one of the important parameters that can help better understand the behavior of nanofluids. In other words, if the effects of particle

migration are appropriately taken into consideration, then more realistic results that are closer to the physics of problem can be attained.

Very few studies have been performed on relationship between particle migration and convective heat transfer of nanofluids. In suspensions containing micro-particles, particles at the wall region tend to move toward the central region, and the near-wall concentration is much lower than the mean concentration. This concentration decrement at the wall side can decrease the effect of the addition of particles, and the thermal conductivity adjacent to the wall does not change sufficiently to enhance the convective heat transfer of a fluid. Therefore, the convective heat transfer coefficient will not change significantly despite the addition of particles with very high thermal conductivity. However, the results published for nanofluids show that addition of nanoparticles can enhance the convective heat transfer considerably. In fact for nanofluids, in addition to factors established for conventional suspensions, other factors play a role in particle migration that can cause different effects as compared to conventional suspensions. Moreover as the particles get smaller, the effect of parameters such as thermophoresis and Brownian motion on particle migration gets more pronounced. This is specifically important for nanofluids which contain nano-scale particles. Thus, particle migration in nanofluids and its effect on the characteristics of flow and heat transfer should be evaluated separately, and the results obtained by studies performed on conventional suspensions cannot be relied upon.

Nanoparticle migration in nanofluids can result in a non-uniform particle distribution, which would change the distributions of thermophysical properties, particularly viscosity and thermal conductivity as they are strong functions of concentration. A reduced viscosity adjacent to the wall could result in the thinning of boundary layer. Nanoparticle migration could also affect the development of boundary layer so as to affect the hydrothermal characteristics. This can lead to serious problem about the validity of the assumption of constant effective properties, as applied by a number of researchers. One should thus be cautious when implementing findings from any simulations based on the uniform property assumption. A proper modeling for the convective heat transfer of nanofluids should implement the two-phase flow nature of nanofluids particularly the migration of nanoparticles and the resultant non-uniform distributions of concentration and effective properties.

4. Theoretical investigations

The study conducted by Ding and Wen [38] in 2005 was the first analytical study on particle migration in nanofluids. They studied particle migration in nanofluids and formulated a theoretical model to predict particle concentration and velocity field in the transverse plane of the pipe for developed laminar flow. The model takes into account the effects of the shear-induced and viscosity gradient-induced particle migrations, as well as self-diffusion due to the Brownian motion. The authors used the following equations to evaluate particle fluxes due to viscosity gradient (J_μ), shear rate (J_c) and Brownian diffusion (J_b).

$$J_\mu = -K_\mu \dot{\gamma}\phi^2 \left(\frac{d_p^2}{\mu}\right) \frac{d\mu}{d\phi}\nabla\phi \tag{1}$$

$$J_c = -K_c d_p^2 \left(\phi^2 \nabla\dot{\gamma} + \phi\dot{\gamma}\nabla\phi\right) \tag{2}$$

$$J_b = -D_b \nabla\phi \tag{3}$$

where K_μ and K_c are constants, $\dot{\gamma}$ is the shear rate, μ represents the viscosity, ϕ denotes the concentration, d_p is the diameter of particles, and D_b represents the Brownian diffusion coefficient calculated by:

$$D_b = \frac{k_b T}{3\pi\mu d_p} \tag{4}$$

where k_b is the Boltzmann's constant, and T denotes the temperature.

Eventually, the following equation was derived to obtain the concentration distribution, which was solved along with Eq. (6).

$$K_\mu \dot{\gamma}\phi^2 \frac{d_p^2}{\mu}\frac{d\mu}{dr} + K_c d_p^2 \phi^2 \frac{d\dot{\gamma}}{dr} + K_c d_p^2 \phi\dot{\gamma}\frac{d\phi}{dr} + D_b \frac{d\phi}{dr} = 0 \tag{5}$$

$$\dot{\gamma} = \frac{r}{2\mu}\left(\frac{dP}{dz}\right) \tag{6}$$

where P represents the pressure, and r and z are the radial and longitudinal coordinates, respectively.

In their investigation, it was revealed that particle concentration in the wall region is much lower than that in the central region. In addition, the particle concentration could only be assumed uniform if the Peclet number is smaller than 10. The reason is that in these Peclet numbers, particles are of a finer size and consequently, the role of Brownian motion in particle migration is considerable. In fact, in contrast to shear rate, Brownian motion tends to uniform the particle distribution.

The Peclet number was defined as below:

$$Pe = \frac{3\pi d_p^3 (-dP/dz) R}{2k_b T} \tag{7}$$

where R represents the radius of the tube.

Physically, the Peclet number shows the ratio of particle migration caused by convection to that resulting from Brownian diffusion. Indeed, shear-induced migration causes the motion of particles from regions of higher shear rate to regions of lower shear rate, viscosity gradient causes particle migration from regions of higher viscosity to those of lower viscosity, and self-diffusion due to the Brownian motion causes particle migration from regions of a greater concentration to those of a lower concentration. The significant non-uniformity is primarily due to the stronger contributions of the shear- and viscosity-induced particle migration than that of the Brownian motion. The net result of the Brownian motion is redistribution of particles between higher concentration regions to lower concentration regions, which displays macroscopically as diffusion. At the limit of $Pe \to 0$, particle concentration will be uniform. Fig. 4 illustrates the particle concentration distribution at a cross section of the tube obtained from the study conducted by Ding and Wen [38] for different Peclet numbers. It can be observed that more non-uniform particle distribution is obtained at the higher Peclet numbers. In fact, Pe number reveals the transition from intensely Brownian to weakly Brownian behavior.

The effects of thermophoresis and inertia on particle migration were not considered in their study.

Employing this approach, Wen and Ding [4] also assessed the effect of particle migration on nanofluid heat transfer under a laminar flow regime in small channels. The results demonstrated a significant non-uniformity in particle concentration and, hence, thermal conductivity over the tube cross section, particularly for large particles at high concentrations. Compared to the constant thermal conductivity, the non-uniform distribution led to a higher Nusselt number, which depended on Peclet number and mean

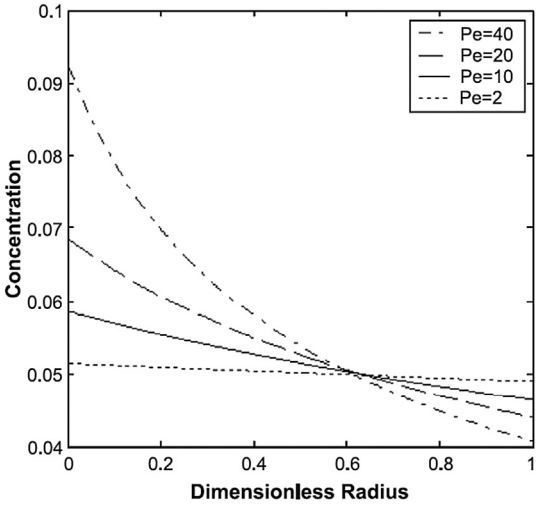

Fig. 4. Particle concentration distribution at a tube cross section for different Peclet numbers [38]. Reprinted with permission from Elsevier.

concentration.

In order to find the diffusion fluxes in Eqs. (1) and (2), the phenomenological constants (i.e. K_c and K_μ) should be determined. Bahiraei [39] utilized the Euler-Lagrange method to evaluate these constants, where gravity, drag, Saffman's lift, Brownian and thermophoretic forces were considered. The results revealed that the constants are not much dependent on concentration, but they change significantly by varying Peclet number. Furthermore, particle migration caused non-uniformity in the distribution of properties and, consequently, reduced thermal conductivity and viscosity adjacent to the wall due to lower concentration there.

Bahiraei and Vasefi [40] investigated the hydrothermal characteristics of nanofluids in a laminar flow inside a tube using the dispersion model. A new model was proposed for dispersion thermal conductivity, and the effects of particle migration were considered to predict the particle distribution at the tube cross section. Like Ding and Wen [38], they considered the effects of non-uniform shear rate, Brownian diffusion and viscosity gradient on particle migration. The obtained particle distribution was applied in the proposed dispersion model. It was observed that nanoparticles are not distributed uniformly at the tube cross section such that the values of concentration are higher at the central regions and this non-uniformity intensifies at higher mean concentrations and Reynolds numbers. The thermal dispersion model that was suggested by them is as below:

$$k_d = c(\rho c_p)\left(\frac{\partial v_x}{\partial r}\right)\phi(r)Rd_p \tag{8}$$

where ρ, v, and c_p represent the density, the velocity, and the specific heat, respectively.

As can be observed in Eq. (8), the non-uniform concentration distribution was applied in the dispersion model.

In addition to the factors considered by Ding and Wen [38], Kang et al. [41] took inertial migration of nanoparticles into consideration as well.

The flux due to inertial migration was written as:

$$J_{IM} = \phi u_{IM} \tag{9}$$

The inertial migration velocity u_{IM} was evaluated via the correlation presented by Ho and Leal [42] as follows:

$$u_{IM} = \frac{\rho V_m^2 d_p^3}{6\pi\mu d^2} G(s) \tag{10}$$

where V_m is maximum velocity, s is the dimensionless coordinate, and function $G(s)$ has been reported by Ho and Leal [42].

They evaluated the distribution of particles and compared the mechanisms for the heat transfer enhancement in a nanofluid with those in a general micro-particle suspension. In comparison with a micro-particle suspension, the authors showed that there is no significant migration in the nanofluid, with no change in particle concentration. It was illustrated that in a nanofluid, particle migration to the center occurs quite slowly. A uniform distribution of particles was observed at Peclet numbers <5. On the other hand, a non-uniform distribution of particles was noticed at a Peclet number of 20. They argued that the non-uniformity observed at high Peclet numbers is mainly due to the stronger contributions of shear-induced and viscosity-induced migration. In addition, the influence of inertial migration decreased by decreasing particle size, and its effect was minor in nanofluids.

Most of the studies which have analytically investigated particle migration in nanofluids have overlooked the effect of thermophoresis. The effect of thermophoresis has been studied analytically by some researchers. Bahiraei and Hosseinalipour [43] evaluated the particle migration in nanofluids considering thermophoresis. They investigated the effects of particle migration on concentration distribution and convective heat transfer of the water-TiO_2 nanofluid inside a circular tube. The concentration distribution was obtained considering thermophoresis, non-uniform shear rate, Brownian diffusion and viscosity gradient. They used the following equation to obtain the concentration distribution:

$$K_\mu \dot{\gamma}\phi^2\left(\frac{d_p^2}{\mu}\right)\frac{d\mu}{d\phi}\frac{d\phi}{dr} + K_c d_p^2\left(\phi^2\frac{d\dot{\gamma}}{dr} + \phi\dot{\gamma}\frac{d\phi}{dr}\right) + D_b\frac{d\phi}{dr} + D_T\frac{dT/dr}{T} = 0 \tag{11}$$

where D_T denotes the thermophoresis coefficient. The last term in Eq. (11) indicates the effect of thermophoresis.

It was found that non-uniformity of the concentration is intensified by increasing the Reynolds number and particle size. Moreover, thermophoresis made the concentration more non-uniform and the velocity profile flatter as well. At greater mean concentrations, the effect of thermophoresis on particle distribution and convective heat transfer increased. Meanwhile, considering thermophoresis yielded a greater convective heat transfer in all Reynolds numbers.

Hwang et al. [44] also took the effect of thermophoresis into consideration. They discussed the effects of thermal conductivity under static and dynamic conditions, energy transfer by nanoparticle dispersion, nanoparticle migration due to viscosity gradient, non-uniform shear rate, Brownian diffusion and thermophoresis on significant enhancement of convective heat transfer of nanofluids. Based on scale analysis and numerical solutions, for the first time, it was demonstrated that the flattened velocity profile due to particle migration and caused by Brownian diffusion and thermophoresis is a possible mechanism of the convective heat transfer enhancement. It was illustrated that thermophoresis as well as Brownian diffusion has important effect on particle migration, and the effects of viscosity gradient and non-uniform shear rate on particle migration can be negligible. This conclusion is in contrast with findings related to conventional suspensions.

The studies mentioned in the above part have investigated the effect of particle migration in laminar flow; however, there are few

studies dealing with particle migration in a turbulent flow regime. Bahiraei [45] evaluated flow and heat transfer characteristics of the suspensions containing Fe_3O_4 magnetic nanoparticles in a turbulent flow regime. The effects of Brownian motion, shear rate and viscosity gradient were taken into account. By applying the effects of particle migration, the amount of concentration at the wall vicinity was lower than that at the tube center. Non-uniformity of the concentration distribution was more significant for the coarser particles and intensified by increasing mean concentration and Reynolds number.

Alongside the abovementioned analytical studies in which concentration distribution is obtained through an equation including various effects on nanoparticle migration (like Eq. (11)), some researchers have studied the effect of particle migration on nanofluids using other analytical methods. Pakravan and Yaghoubi [46] investigated theoretically the basics of natural convection heat transfer of nanofluids considering thermophoresis and Dufour effect. To find the concentration distribution, an equation in addition to the conservation equations was written:

$$\nabla\cdot\left(\rho_p\phi\mathbf{v}\right)=-\nabla\cdot\mathbf{j}_p \tag{12}$$

where subscript p refers to the particles, and $\mathbf{j}_p$ represents the flux of particles that was considered as a combination of the effects of thermophoresis and Brownian motion:

$$\mathbf{j}_p=-D_b\nabla\phi-D_T\nabla T \tag{13}$$

Similar to the particle flux, diffusive heat flux was assumed as a combination of the effects of temperature gradient (conduction) and volume fraction gradient (Dufour):

$$\mathbf{j}_h=-D_C\nabla T-D_D\nabla\phi \tag{14}$$

where D_D denotes the Dufour coefficient and D_C is the conduction coefficient.

The authors, in their theoretical approach, approximated the mixture an ideal system due to low volume fraction. They also assumed one-dimensional field and a thin layer of nanofluid, to approximate the gradients with their differences. Furthermore, to find the Dufour coefficient, the Onsager reciprocal relationship was employed. For approximate conditions, closed form expression was derived to predict the nanofluid Nusselt number based on the base fluid Nusselt number as below:

$$Nu=Nu_f-3\pi S_T^2\frac{\phi}{1-\phi}\left(\frac{\rho_p}{\rho_f}\right)^2\frac{R}{k_b}\frac{\mu^3}{\rho_p}\frac{d_p}{k} \tag{15}$$

where subscript f refers to the base fluid, k is the thermal conductivity, and S_T represents the thermophoresis parameter.

It was demonstrated that the thermophoresis makes nanoparticle distribution non-uniform and this non-uniformity induces another heat transfer opposing the direction of temperature decrease and hence Nusselt number decreases due to the combined effects of thermophoresis and Dufour processes.

Peng et al. [47] proposed a model for predicting the migration characteristics of nanoparticles during the refrigerant-based nanofluid pool boiling. In establishing the model, the departure and rising processes of bubble, as well as movement of nanoparticles in liquid-phase were firstly simulated; then the capture of nanoparticles by bubble and escape of nanoparticles from the liquid-vapor interface were simulated; finally, the migration ratio of nanoparticles was obtained by flotation theory combining the analysis on the boiling process (see Fig. 5). The proposed model can predict the influences of nanoparticle size, refrigerant type, mass fraction of lubricating oil, heat flux, and initial liquid-level height on the migration of nanoparticles. It was claimed that the migration ratio of nanoparticles predicted by the model agrees with the experimental data.

Jianzhong et al. [48] investigated nanoparticle migration in a fully developed pipe flow. The evolution of particle number concentration, total particle mass, polydispersity, particle diameter and geometric standard deviation were obtained by using a moment method to approximate the particle general dynamic equation. In their approach, the nanoparticles undergo diffusion and coagulation in a turbulent flow, and their clustering is governed by the general dynamic population balance equation:

$$\frac{\partial n}{\partial t}+u_j\frac{\partial n}{\partial x_j}=\frac{\partial}{\partial x_j}\left(D\frac{\partial n}{\partial x_j}\right)+\frac{1}{2}\int_0^{v}\beta(v-\bar{v},\bar{v})n(v-\bar{v},\mathbf{x},t)n(\bar{v},\mathbf{x},t)d\bar{v}-n(v,\mathbf{x},t)\int_0^{\infty}\beta(v,\bar{v})n(\bar{v},\mathbf{x},t)d\bar{v} \tag{16}$$

where $n=n(v,\boldsymbol{x},t)$ denotes the particle size distribution function based on cluster volume, u_j is the flow velocity, D represents the coefficient of diffusivity, β is the collision coefficient which is dependent on particle size, v and $\bar{v}$ are the two classes of particles of volumes. On the left-hand side of Eq. (16), the first term is the time rate of change of particle concentration and the second term represents the fluid transport. On the right-hand side, the first term is the particle diffusion, the second term represents the production rate of particles of volume v by collision of particles of volumes $v-\bar{v}$ and $\bar{v}$, and the third term denotes the disappearance rate of particle cluster having volume v by collisions with particles of all sizes. The results showed that nanoparticles move to the pipe center. Moreover, the particle number concentration and total particle mass were distributed non-uniformly, and the largest particle clusters were found in the pipe center. The authors claimed that particle migration for nanoparticles is different from that for micro-particles.

Giraldo et al. [49] conducted an investigation using boundary integral equations to simulate particle interactions. By adapting the formulation presented in Refs. [50], they obtained a fully implicit model for the motion of nanoparticles in the vicinity of a wall that takes into consideration the effects of viscous drag from the velocity field and that created as a response to the forces acting on the particles. Factors such as Brownian motion, van der Waals forces and electrical double layer interaction were considered. The results showed that a zone with about 17% higher concentration was created about 0.3 μm away from the wall; this can increase the thermal conductivity and, thus, heat transfer.

The result obtained in most analytical studies indicates that the concentration distribution resulting from particle migration depends considerably on parameters such as mean particle concentration, size of particles, Peclet number, and Reynolds number. In a large number of analytical studies, the utilized correlations require knowing empirical constants. Conducting experimental studies for observing particle motion at nano-scale would prove to be quite complicated; thus, constants obtained for micro-particles are used for these correlations. This, however, can lead to errors in predicted results. In addition, a review of analytical surveys performed about particle migration in nanofluids reveals that most authors had to use many simplifying assumptions for their modeling due to complexity of phenomena. In future, more comprehensive models including factors such as entrance effects, the dynamics of particles, particle–wall interactions, and so forth, for analytical studies on nanoparticle migration are required.

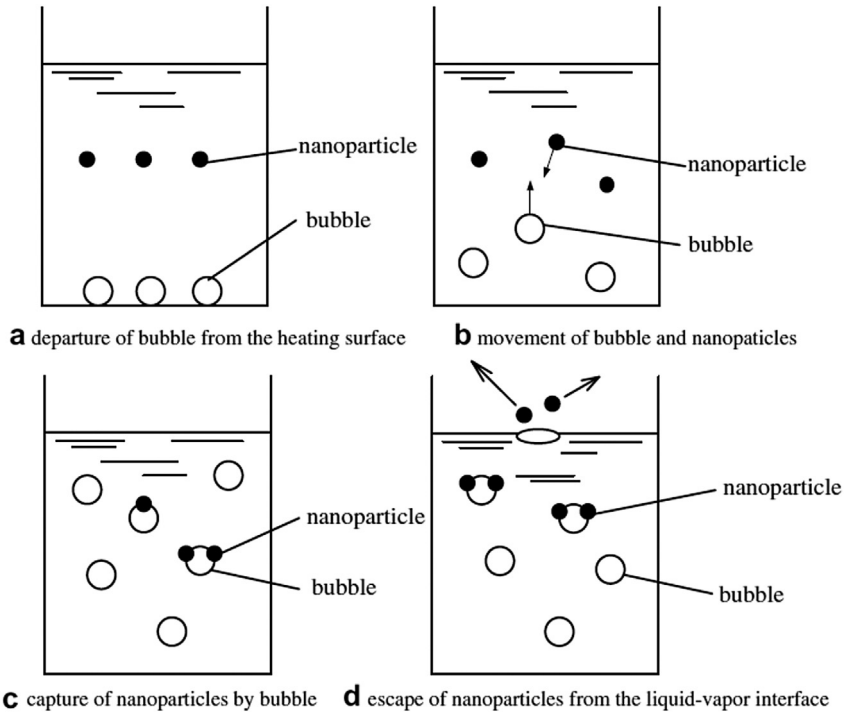

Fig. 5. Schematic of the migration process of nanoparticles from liquid-phase to vapor-phase [47]. Reprinted with permission from Elsevier.

5. Numerical investigations

Some researchers have assessed particle migration using various numerical methods. The most important numerical approaches employed for this purpose are the Eulerian-Lagrangian approach and the Buongiorno four-equation model [51].

5.1. Eulerian-Lagrangian approach

Most of the studies conducted on nanofluids have employed the homogenous (single-phase) simulation method. In the single-phase approach, conservation equations are solved taking into account the effective properties. Indeed, due to the extremely fine size of particles, it is assumed that the thermal and hydrodynamic equilibriums exist between nanoparticles and base fluid, and consequently, nanofluids are considered as a single fluid.

Although the validation of single-phase modeling with the experimental data is approximately in good agreement, this approach cannot evaluate the fluid flow and thermal features induced by particle migration effects. This incapability of the single-phase technique particularly while investigating thermal and fluid flow behavior of nanofluids, makes the Eulerian-Lagrangian approach a more reliable one. Indeed, the slip velocity between the fluid and nanoparticles may not be zero due to several factors such as gravity, friction between the fluid and particles, Brownian diffusion, sedimentation and dispersion.

In the Eulerian-Lagrangian approach, the dependence on effective models of thermal conductivity and viscosity is completely eliminated, in which the individual properties of the fluid and nanoparticles are only required. The Eulerian-Lagrangian approach involves tracking of particles in the domain, thereby giving more understanding about hydrodynamics of particles. This approach provides a field description of the dynamics of each phase, and Lagrangian trajectories of individual particles coupled with Eulerian description of fluid flow field [52,53].

By decreasing the particle size, in addition to drag, gravity and pressure forces that apply on a particle, other forces such as Brownian and thermophoretic that can be neglected for large particles will have substantial roles. For instance, for ultrafine particles, the instantaneous momentum imparted to the particle fluctuates randomly, which causes the particle to move on a chaotic path known as Brownian motion. Saffman lift force is also significant for particles traveling through a strong shear flow. Effective forces on nanoparticle motion such as Brownian, thermophoretic, gravity, pressure, virtual mass, drag, Saffman lift, and so forth are taken into consideration in the Eulerian-Lagrangian approach. In this method, the fluid is considered as a continuous phase with nanoparticles dispersed inside it. For the continuous phase, mass, momentum and energy equations are the governing equations of fluid flow and heat transfer. The motion of nanoparticles is governed by the Newton's second law in the Lagrangian frame. The discrete and continuous phases are coupled via Newton's third law. The sum of all forces applying on the particle is reflected in the continuous phase through the momentum source term S_p in Eq. (18). In fact, S_p updates based on momentum change that occurs in the control volume caused by the motion of nanoparticles. Similarly the heat transfer in the particle phase is reflected via the energy source term S_t in Eq. (19). This source term is simply computed from the energy balance within a computational cell.

The main difference between the Eulerian-Lagrangian approach and the one utilized by Ding and Wen [38], which was investigated in the previous section, is the fact that, in the model of Ding and Wen [38], in addition to base fluid, the dispersed particles are also treated as continuous phase.

In the Eulerian-Lagrangian approach, the conservative equations for the continuous phase would be written as below:

Continuity equation:

$$\nabla \cdot \left(\rho_f \mathbf{v}_f \right) = 0 \tag{17}$$

Momentum equation:

$$\nabla\cdot\left(\rho_f\mathbf{v}_f\mathbf{v}_f\right)=-\nabla P+\nabla\cdot\left(\mu_f\nabla\mathbf{v}_f\right)+\mathbf{S}_p \tag{18}$$

Energy equation:

$$\nabla\cdot\left(\rho_f c_{p,f}\mathbf{v}_f T_f\right)=\nabla\cdot\left(k_f\nabla T_f\right)+S_t \tag{19}$$

An essential condition for using the Eulerian-Lagrangian approach is the possibility to consider the liquid phase as a continuous medium for the motion of nanoparticles. To make sure the above condition is met, the Knudsen number should be calculated. If the Knudsen number is small enough, then the condition of continuity of liquid phase is reliable. Knudsen number is the ratio of mean free path of fluid molecules to the diameter of particle. For instance, the mean free path for water is about 0.3 nm and the diameter of nanoparticles is between 1 and 100 nm. Therefore, the fluid phase around the nanoparticles can be considered as a continuum since the Knudsen number is rather small (Kn < 0.3).

In the studies that have employed the Eulerian-Lagrangian approach for nanofluids, the following equations have often been used for calculating the related forces.

Drag force [54]:

$$\mathbf{F}_D=\frac{18\mu_f}{d_p^2\rho_p C_c}\left(\mathbf{v}_f-\mathbf{v}_p\right) \tag{20}$$

where C_c is Cunningham correction factor to Stokes' drag law which can be extracted from:

$$C_c=1+\frac{2\lambda}{d_p}\left(1.257+0.4\exp\left(-1.1d_p/2\lambda\right)\right) \tag{21}$$

where λ represents the molecular mean free path.

Saffman lift force [55]:

$$\mathbf{F}_L=\frac{2K_s\nu^{1/2}\rho_f d_{ij}}{\rho_p d_p\left(d_{ij}d_{ij}\right)^{1/4}}\left(\mathbf{v}_f-\mathbf{v}_p\right) \tag{22}$$

where $K_s=2.594$ and d_{ij} is tensor of deformation. This form of the lift force is recommended for small particle Reynolds numbers.

Thermophoretic force [56]:

$$\mathbf{F}_T=-6\pi\mu_f^2 d_p C_s\frac{1}{\rho_f(1+3C_mKn)}\frac{k_f/k_p+C_tKn}{1+2k_f/k_p+2C_tKn}\frac{\nabla T}{m_pT} \tag{23}$$

where m_p denotes the particle mass, and $C_s=1.17$, $C_m=1.14$ and $C_t=2.18$.

Brownian force:

$$F_{Bi}=\zeta_i\left(\frac{\pi S_0}{\Delta t}\right)^{1/2} \tag{24}$$

where ζ_i is the unit-variance-independent Gaussian random number with zero-mean. Different components of the Brownian force are modeled as a Gaussian white noise process, while the spectral intensity of $S_{n,ij}$ is given by Li and Ahmadi [57]:

$$S_{n,ij}=S_0\delta_{ij} \tag{25}$$

where δ_{ij} is the Kronecker delta function, and

$$S_0=\frac{216\nu k_b T}{\pi^2\rho_f d_p^5\left(\frac{\rho_p}{\rho_f}\right)^2 C_c} \tag{26}$$

where ν represents the kinematic viscosity.

He et al. [58] and Bianco et al. [59] were pioneers who used the Eulerian-Lagrangian approach for nanofluids, and modeled nanoparticles as discrete phase. The former utilized the particle interaction source term only in momentum equation; however, the latter employed both momentum and energy source terms in governing equations.

Some researchers have compared the results obtained through the Eulerian-Lagrangian approach with single-phase method, and have shown that the Eulerian-Lagrangian approach yields results closer to experimental data. Singh et al. [60] evaluated the nanofluid flow in microchannels experimentally and numerically. The hydrodynamic, body, Brownian and thermophoretic forces were taken into account. It was found that the Eulerian-Lagrangian approach is more accurate as compared to the single-phase model. The results also showed non-uniformity of nanoparticle distribution across the channel cross-section. The authors claimed that this non-uniformity can be attributed to the shear induced particle migration and can also be the reason for the difference in pressure drop and heat transfer from the single-phase model.

Jacob et al. [61] studied the behavior of Al_2O_3-water nanofluid in natural convection induced by heat generation using microwave heating by single-phase and Eulerian-Lagrangian approaches. This study demonstrated that additional flow in nanofluids is induced by the heated nanoparticles due to particle migration. Furthermore, Brownian, thermophoretic and Saffman's lift forces were found to be negligible and the buoyant force was the dominant one. The transient variations of isotherms obtained by single-phase and Eulerian-Lagrangian approaches were different. The authors argued that the single-phase approach cannot capture the behavior induced by particle migration, and flow patterns in nanofluids get modified due to particle migration.

Generally speaking, one of the main reasons for the difference observed between homogenous and Eulerian-Lagrangian approaches can be caused by particle migration which occurs during flow of nanofluids.

Some studies, using the Eulerian-Lagrangian approach, have evaluated particle migration and its effect on the hydrothermal characteristics of nanofluids. A numerical simulation based on the Eulerian-Lagrangian approach was carried out by Wen et al. [62] to investigate the flow and migration of nanoparticles in a single channel. The interparticle forces such as the van der Waals and electrostatic forces were neglected due to their relatively small contributions. Moreover, no particle agglomeration was considered and particle collisions were also neglected. It was illustrated that a significant non-uniform concentration occurs in the transverse direction, being higher in the tube center. Moreover, the smaller the particle size, the more uniform the distribution of particles.

Rostami and Abbassi [52] studied nanofluid conjugate heat transfer in wavy microchannels using the Eulerian–Lagrangian approach. The nanofluid was water–Al_2O_3 with $d_p=120$ nm and volume fraction from 0% to 2%. The Nusselt number increased because of wavy walls of the microchannel in compare with straight walls microchannels and because of using nanofluid instead of water as working fluid. Also, it was found that the distribution of particles was not homogeneous and was space-dependent.

Bahiraei and Hangi [63] evaluated the energy efficiency of the water-Al_2O_3 nanofluid in a C-shaped chaotic channel as well as a

straight one using the Eulerian-Lagrangian method. It was illustrated that the concentration distribution is non-uniform at the cross section of the straight channel, while intense mixing in the C-shaped channel makes distribution of the nanoparticles uniform. The authors claimed that simultaneous application of nanofluids, as heat transfer fluids, and chaotic channel, as a modified geometry, can result in not only higher energy efficiency, but also preventing nanoparticles agglomeration due to intense mixing.

Mahdavi et al. [64] investigated hydrothermal features of a nanofluid within a vertical tube through the Eulerian-Lagrangian approach. Three common types of nanofluids consisting of alumina, zirconia and silica nanoparticles were studied. The velocity profiles showed that the slip velocity between nanoparticles and base fluid was not negligible. The authors proved that nanoparticles migrate from the wall both radially and tangentially. Also, it was shown that gravity cannot be neglected and absence of gravity force underestimates pressure drop. Non-dimensional temperature distributions of fluid and nanoparticles at the tube outlet obtained from this work are depicted in Fig. 6 for alumina nanofluids. It was defined as:

$$T^* = \frac{2(T - T_{in})k}{q'' D} \tag{27}$$

where q'' and D denote the heat flux and the inside diameter of the tube, respectively.

As per Fig. 6, it is noticed that the base fluid temperature has a smooth parabolic profile. It can be stated that nanoparticles cannot distort the parabolic shape of the temperature profile, and energy is mostly transported via nanoparticle migration. This makes more sense when fluid temperature is compared with the distribution of nanoparticle temperature in this figure. The nanoparticle temperature profiles are distorted at the wall vicinity because of the greater temperature gradient. Moreover, the trend is almost similar for lower and higher concentrations.

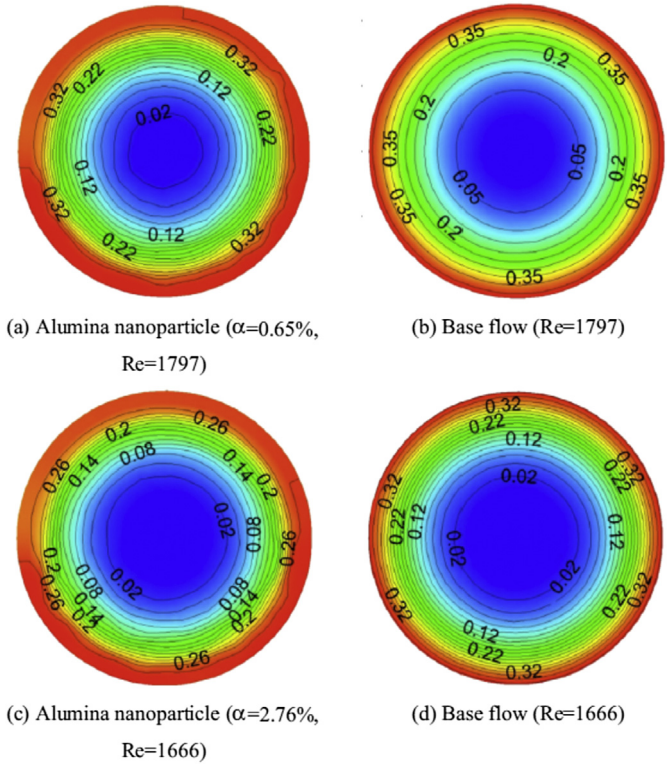

Fig. 6. Non-dimensional temperature distributions [64]. Reprinted with permission from Elsevier.

Bianco et al. [59] investigated developing forced convection of a nanofluid in a tube under a uniform wall heat flux using the Eulerian-Lagrangian approach. For drag force, they utilized the Stokes law to specify the drag coefficient. Heat transfer enhancement increased with particle concentration, but it was accompanied by increasing wall shear stress. It was concluded that as thermal entrance length is dependent on Prandtl number, when concentration increases Pr number also increases and consequently, thermal entrance length becomes greater.

He et al. [58] performed a numerical study by using the Euler-Lagrange method on the convective heat transfer of the water-TiO_2 nanofluid flowing through a tube. It was concluded that the results deviate from the experimental data for small x/D values particularly for the higher Reynolds number. Furthermore, simulations were carried out in the presence and absence of the Brownian and thermophoretic forces and it was illustrated that these forces have a little effect on heat transfer.

Turbulent flow in helically coiled tubes under constant wall heat flux was numerically and experimentally investigated by Bahremand et al. [65]. The numerical computations were performed by the Eulerian-Lagrangian approach in connection with the RNG $k-\varepsilon$ turbulence model accounting for four-way coupling collisions. The results showed that the nanoparticles do not change the axial velocity and turbulent kinetic energy significantly, while the microparticles increase axial velocity and suppress turbulence. It was found that by decreasing particle size, the movement of particles with turbulence eddies becomes easier. Therefore, the particle enlargement reduced turbulent kinetic energy.

Tahir and Mital [66] presented numerical investigation of developing laminar forced convection of the alumina-water nanofluid subjected to a uniform wall heat flux. They used the Eulerian-Lagrangian approach for simulation. It was found that the heat transfer coefficient linearly increases with both Reynolds number and volume fraction, but shows non-linear parabolic decrease with an increase in particle size. The authors indicated that the model equation can be used to optimize the performance of nanofluids for use in miniature liquid-cooled heat sinks.

Su et al. [67] evaluated magnetic nanofluid transport using agarose gel. They developed a particle tracking model to study the migration and deposition of nanoparticles in the porous structure under multiple forces including Brownian motion, London-van der Waals attraction, electrostatic forces, gravitational force, viscous force, and inertial force. Their model allows for determination of the rate of nanoparticle deposition on the porous structure. The authors claimed that the information obtained can be used with continuous porous medium theory to predict the evolution of the concentration and deposition profiles of nanoparticles in porous structure.

One of the main challenges in the Eulerian-Lagrangian approach is the reliability of correlations used for forces applied on nanoparticles. It is noteworthy that the formats of most of the forces that have thus far been used in the Eulerian-Lagrangian approach for nanofluids have been established for relatively large particles. They may not be applicable to nanoparticles due to, for example, the rarefaction, and it is not obvious currently how these correlations are corrected for nanoparticles. Some researchers have tried to introduce new forms of forces for investigating particle migration in nanofluids.

One of important forces in studies conducted on nanofluids is thermophoretic force. A brief review of thermophoresis in gases reveals that the hydrodynamic theory of thermophoresis in liquids is an extension of that in gases. Contrary to thermophoresis in liquids, which is rather new and is scarcely understood,

thermophoresis in gases has been the subject of study for approximately 100 years. Despite gases in which thermophoresis has been properly evaluated and documented, thermophoresis in liquids has been poorly formulated. Fayolle et al. [68] studied thermally induced particle flow in a charged colloidal suspension by a fluid-mechanical approach. From Stokes' equation with no-slip boundary conditions at the particle surface, they presented the thermophoretic transport coefficients.

Epstein [69] developed an equation for the thermophoretic velocity of particles suspended in ideal gases. Later Talbot et al. [56] performed experiments on thermophoresis in heated boundary layers. The expression proposed by Talbot et al. [56] is applicable to gases only, but erroneously has been applied by many researchers for thermophoresis in nanofluids. This can lead to errors in results.

Some other researchers have used the equation introduced by McNab and Meisen [70] so as to take thermophoresis into consideration. However, this model has been obtained for particle sizes greater than 1 μm. In spite of the criticism voiced against the credibility of the McNab-Meisen experimental data, e.g. Refs. [71,72], this equation has been extensively utilized to predict thermophoresis in nanofluids. Due to the presence of non-continuum effects in thermophoresis of nanoparticles, it is expected that thermophoresis for nanoparticles be dissimilar from micro-particles.

Some investigators have made an attempt to present a correct form for considering thermophoresis in nanofluids. In the study conducted by Eslamian and Ziad Saghir [73], a non-equilibrium thermodynamic based expression was developed to estimate thermophoretic velocity in nanofluids. The results suggested that the general form of this equation is valid for thermophoresis of nano-sized and even sub-nanometer particles in liquids; however, the correct prediction of the matching parameter is still unresolved. Also, the non-equilibrium thermodynamics combined with the concept of activation energy of viscous is somewhat capable of estimating thermophoresis coefficient of inert particles and macromolecules of about 1 nm or smaller. The focus of this work was on thermophoresis in liquids. The findings showed that the expressions of McNab and Meisen [70] and Talbot [56] highly overestimate the thermophoresis mobility. It is known that these two expressions are presently utilized to model thermophoresis in nanofluids.

The other important force that is considered in the simulation of nanofluids through the Eulerian-Lagrangian approach is the Brownian force. In the simulations on the movement of nanoparticles implemented by many researchers, the applied Brownian force model is the one developed by Li and Ahmadi [57] (Eq. (24)), which is based on the assumption that the mean square displacement increases linearly with time. Few attempts have been made to present new correlations for the Brownian force in nanofluids. Dong et al. [74] proposed a new expression to simulate Brownian force based on the experimental measurements, which follows white Gaussian noise process. As the time $t \rightarrow 0$ and the particle density is equal to the fluid density, the new expression approaches the classical formula of the model used by many researchers. The present model was applied in simulating flow and heat transfer in a channel utilizing alumina–water nanofluid. The results showed that the distribution of nanoparticles inside the channel is obviously unsteady and non-uniform. Moreover, the profiles of velocity and temperature showed significant fluctuations at low Reynolds numbers.

Drag force is the other important force which is taken into consideration in the Eulerian-Lagrangian approach. Although Stokes-Cunningham drag law (Eq. (20)) has been proposed by Ounis et al. [54] for submicron particles, a large number of researchers have utilized spherical drag law for nanofluids, which is applicable only for microparticles.

The other point that needs to be taken into consideration is that in the analyses conducted on nanofluids in which the Eulerian-Lagrangian approach has been utilized, the turbulence effects of flow on particle tracking in turbulent flow regime has not received due attention. Unlike particle tracking in laminar flow, tracking particles in turbulent flow requires consideration of turbulent dispersion of particles. To better address the effect of chaotic nature of turbulence, the effect of instantaneous turbulent velocity fluctuations on trajectories of nanoparticles should be considered. Future studies should adopt unsteady tracking in the Lagrangian domain, or even conduct transient simulations in both Eulerian and Lagrangian domains. The reason is that turbulence is time dependent and by transient simulation, the flow features are considered as a function of time. Moreover, a Large Eddy Simulation (LES) is proposed, because it can provide a view of how particles interact with large eddies.

The other issue in the Eulerian-Lagrangian simulations conducted on nanofluids is that the interactions of nanoparticles have been overlooked. In nanofluids, nanoparticles have a random thermal motion. Therefore, nanofluids are dynamical mediums and their thermal characteristics depend not only on the nanostructures in these suspensions but also the dynamics of nanoparticles. Indeed, there are interactions between nanoparticles or between nanoparticles and liquid molecules, should be taken into account to develop more realistic models.

The mean distance between the nanoparticles in nanofluids is not that great. Therefore, the van der Waals force and electrostatic repulsion force between the nanoparticles can be considerable. Strong interactions can restrict fluctuations of the nanoparticles, and vary the effect of micro-mixing on heat transfer.

Employing order-of-magnitude analysis, Kleinstreuer and Feng [75] showed that interactions between the nanoparticles, such as van der Waals force and electrostatic repulsion force, should be considered even for dilute nanofluids. The electrostatic repulsion force, which is a short-distance force, originates from the overlap of the electric double layers around the nanoparticles when they become close to each other. In fact, the energy redistribution resulting from nanoparticle interactions may contribute to the enhancement of heat exchange.

Giraldo et al. [76] investigated the flow and thermal behavior of a nanofluid containing alumina nanoparticles. They considered the effects of particle–particle and particle–fluid interactions. It was concluded that particle-particle interactions cause a change in the velocity and temperature profiles, and consequently, a change in heat transfer rate.

Vladkov and Barrat [77] modeled the thermal properties of a nanofluid considering the effects of particle-particle and particle-fluid interactions. In order to increase the rate of particle-particle energy transfer, the Lennard-Jones interaction intensity between the nanoparticles was varied. It was found that the temperature difference between the particles decreases by a factor of two when the interaction intensity is increased by a factor of ten. Moreover, the thermal conductivity was greater for the strong attractive interactions.

Particle-particle interaction should be considered for the Eulerian-Lagrangian approach in future investigations, particularly in great concentrations. This is because when particle concentration is great, particles might collide more and the effect of particle collision should not be overlooked. In addition, it should be noted that, in practice, nanoparticles could form structures such as clusters or become agglomerated that affects particle migration significantly.

Indeed, the Eulerian-Lagrangian approach can be weak in simulation of nanofluids with high nanoparticle loading due to the

inclusion of phenomena such as agglomeration. Generally, the conventional Eulerian-Lagrangian frame, which is usually applied, can be insufficient for nanoparticles because of their extremely small sizes, and therefore, molecular dynamic simulation, Brownian dynamic simulation or solution of Boltzmann transport equation are necessary. These mentioned alternatives can help to better understand the migration behavior in nanofluids.

5.2. Studying particle migration via Buongiorno model

Another attempt for considering the effect of particle migration and reaching more realistic results for nanofluid characteristics was made by Buongiorno [51]. He suggested a mathematical model for nanofluids by taking into account the Brownian motion and thermophoresis. Investigating the nanoparticle migration, Buongiorno [51] considered seven slip mechanisms such as inertia, Brownian diffusion, thermophoresis, diffusiophoresis, Magnus effect, fluid drainage, and gravity. He concluded that of those seven only Brownian diffusion and thermophoresis are important slip mechanisms. Based on this result, Buongiorno developed a two-component four-equation nonhomogeneous equilibrium model for mass, momentum, and heat transport in nanofluids. The nanofluid was treated as a two-component mixture with the following assumptions:

1 Incompressible flow,
2 No chemical reactions,
3 Negligible external forces,
4 Dilute mixture,
5 Negligible viscous dissipation,
6 Negligible radiation heat transfer,
7 Thermal equilibrium between nanoparticles and base fluid.

Under the above assumptions, the four equations including nanofluid continuity, nanoparticle continuity, nanofluid momentum, and nanofluid energy were written for a steady, two-dimensional laminar flow as below:

Nanofluid continuity equation:

$$\nabla\cdot\mathbf{v}=0 \tag{28}$$

Nanoparticle continuity equation:

$$\frac{\partial\phi}{\partial t}+\mathbf{v}\cdot\nabla\phi=\nabla\cdot\left[D_b\nabla\phi+D_T\frac{\nabla T}{T}\right] \tag{29}$$

Eq. (29) shows that nanoparticles not only can move homogeneously with the fluid (second term of the left-hand side), but also have a slip velocity relative to the fluid (right-hand side), which is due to Brownian diffusion and thermophoresis.

Nanofluid momentum equation:

$$\rho\left[\frac{\partial\mathbf{v}}{\partial t}+\mathbf{v}\cdot\nabla\mathbf{v}\right]=-\nabla P-\nabla\cdot\tau \tag{30}$$

Note that Eq. (30) is similar to the momentum equation for a pure fluid. The stress tensor τ can be evaluated assuming the Newtonian behavior and incompressible flow:

$$\tau=-\mu\left[\nabla\mathbf{v}+(\nabla\mathbf{v})^t\right] \tag{31}$$

where the superscript t refers to the transpose of $\nabla\mathbf{v}$.

If viscosity is constant, Eq. (30) becomes the common Navier-Stokes equation. However, viscosity significantly depends on ϕ and therefore, Eqs. (29) and (30) are coupled.

Nanofluid energy equation:

$$\rho c_p\left[\frac{\partial T}{\partial t}+\mathbf{v}\cdot\nabla T\right]=\nabla\cdot k\nabla T+\rho_p c_{p,p}\left[D_b\nabla\phi\cdot\nabla T+D_T\frac{\nabla T\cdot\nabla T}{T}\right] \tag{32}$$

Eq. (32) indicates that heat may be transported via convection (second term on the left-hand side), via conduction (first term on the right-hand side), and also via virtue of nanoparticle diffusion (second and third terms on the right-hand side). The last two terms on the right-hand side explain the additional contribution caused by nanoparticle migration. It should be noted that the conservation equations are strongly coupled. Indeed, $\mathbf{v}$ depends on ϕ via viscosity; ϕ depends on T mostly owing to thermophoresis; T depends on ϕ via thermal conductivity and also via the Brownian and thermophoretic terms in the energy equation; ϕ and T clearly depend on $\mathbf{v}$ owing to the convection terms in the nanoparticle continuity equation and energy equation, respectively.

The difference of Buongiorno model with single-phase traditional ones is the existence of the nanoparticle continuity equation, i.e., Eq. (29); as mentioned before, this equation states that nanoparticles can, as a result of thermophoresis and Brownian diffusions, have slip velocity relative to base fluid. This slip phenomenon makes a non-uniform particle distribution which results in the non-uniform distributions of thermal conductivity and viscosity, and can decrease thermal boundary layer thickness.

In fact, Buongiorno developed this model to clarify the anomalous convective heat transfer in nanofluids and so eliminate the limitations of the homogeneous and dispersion models. He proclaimed that the anomalous heat transfer occurs due to particle migration. The Buongiorno model has been applied by several researchers and for different issues such as Tzou [78] for the study of nanofluid Bernard convection, Kuznetsov and Nield [79] to evaluate the effect of nanoparticles on the natural convection boundary layer flow past a vertical plate, Hwang et al. [44] for the investigation of laminar forced convection, and Nield and Kuznetsov [80] for the evaluation of thermal instability in a porous medium layer saturated by nanofluids.

Schio et al. [81] investigated a nanofluid laminar forced convection in a parallel–plane channel. They solved the fully–elliptic coupled equations proposed by Buongiorno in order to assess the thermal behavior of the nanofluid. Two sample cases were investigated in detail: a linearly changing wall temperature, and a sinusoidally changing wall temperature. The analysis showed that if a linearly varying boundary temperature is assumed, the concentration field depends very weakly on temperature distribution. On the other hand, in case of a longitudinally periodic boundary temperature, non-homogeneity in the concentration distribution arises, which is wrongly neglected in the homogeneous model. Eventually, the authors claimed that thermophoresis and Brownian diffusion may display non-negligible, or even important, effects in realistic cases.

Sheremet et al. [82] presented natural convection heat transfer in a porous enclosure filled with a nanofluid using the model proposed by Buongiorno. The governing equations were solved by finite difference method. It was found that low Rayleigh and Lewis numbers and high thermophoresis parameter reflect essential non-homogeneous distribution of nanoparticles inside the cavity.

Bahiraei et al. [83] investigated the particle migration effects on nanofluid heat transfer via Buongiorno model considering Brownian and thermophoretic forces. The results showed that at greater volume fractions, the effect of wall heat flux change is more significant on nanofluid heat transfer, whereas this effect decreases at higher Reynolds numbers. In addition, the average convective heat transfer coefficient raised by increasing the Reynolds number and volume fraction. Considering the particle migration effects, higher

heat transfer coefficient was obtained and also the concentration at the tube center was higher in comparison with the wall vicinity.

Elshehabey et al. [84] carried out a numerical investigation to study Buongiorno model for MHD mixed convection of a cavity filled with nanofluid. A sinusoidal temperature on both vertical sides was considered, and the horizontal walls were kept adiabatic. The results demonstrated that the presence of an inclined magnetic field leads to the loss of fluid movement. Moreover, the fluid flow was dominated by the movement of the upper wall in the case of the highest values of the buoyancy ratio.

Maghrebi et al. [85] evaluated the effects of migration of nanoparticles on heat transfer in a channel occupied with a porous medium. The nanofluid was treated as a two-component mixture as discussed by Buongiorno. It was presumed that nanoparticles are distributed non-uniformly inside the channel. The concentration distribution did not change significantly with variation of Lewis number. Moreover, increasing the thermophoretic parameter led to an increase in volume fraction of particles, and also caused particles to migrate to the channel center.

Qasim et al. [86] investigated heat transfer and mass diffusion in nanofluid over a permeable moving surface via Buongiorno model. The surface exhibited convective boundary conditions and constant mass diffusion. The shooting technique was implemented for the numerical solution. It was found that an increase in Brownian motion parameter increases the concentration in boundary layer whereas an increase in thermophoresis parameter causes a decrease in the concentration.

Kozlova and Ryzhkov [87] assessed laminar convective heat transfer of water-alumina nanofluid in a tube under uniform wall heat flux. The investigation was performed numerically on the basis of Buongiorno model. The influence of nanoparticle migration on the heat transfer was analyzed comparatively. They suggested the following expression for thermophoretic mobility, and the intensity of thermophoresis was characterized by this new empirical model:

$$D_T = \alpha \frac{\beta_T}{\mu_f} \frac{k_f}{2k_f + k_p} \tag{33}$$

where β_T denotes the thermal expansion coefficient, and α is the proportionality coefficient which is chosen such that the order of D_T must correspond to the experimental data.

The authors argued that their proposed model for thermophoresis presents more precise results compared to the previous model. It was concluded that the effect of thermophoresis on heat transfer is rather weak since concentration varied only in the thin boundary layer near the wall.

Bahiraei et al. [88] evaluated the hydrothermal characteristics of the water-TiO_2 nanofluid within an annulus considering the effects of particle migration. The Buongiorno model was applied and the four coupled equations were numerically solved. The convective heat transfer coefficients increased at both inner and outer walls of the annulus by raising the concentration. Moreover, along the annulus, the friction coefficient decreased more rapidly at lower Reynolds numbers. Taking particle migration into account, a non-uniform concentration distribution was observed at the annulus cross section; higher heat transfer coefficients were obtained at both walls; and the velocity profile became flatter. In addition, the effect of thermophoresis on the convective heat transfer proved to be more significant than that of Brownian diffusion.

Sheremet et al. [89] performed a numerical investigation on the unsteady natural convection of a water based nanofluid within a wavy-walled cavity under the influence of a uniform inclined magnetic field using the model proposed by Buongiorno. The left vertical wavy and right vertical flat walls of the cavity were kept at constant but different temperatures whereas the top and bottom horizontal walls were adiabatic. Isoconcentrations, illustrated in this research, revealed that distribution of nanoparticles is non-homogeneous for the developed heat conduction regime.

A numerical study was conducted by Garoosi et al. [90] using Buongiorno model concerning natural and mixed convection heat transfer of nanofluid in a two-dimensional square cavity with several pairs of heat source-sinks. The configuration of the problem and locations of heat source-sinks along with streamlines, isotherms and contour of nanoparticle distribution have been illustrated in Fig. 7 for $Ra = 10^5$. It was found that at high Richardson numbers and low Rayleigh numbers, the particle distribution is fairly non-uniform. Moreover, the thermophoretic effects were negligible for nanoparticles with high thermal conductivity. The authors argued that in such conditions, the use of single-phase models is valid at any Rayleigh and Richardson numbers.

Some investigations dealt with finding analytical solution for Buongiorno equations through Homotopy Analysis Method (HAM). HAM is a robust analytic approach to solve ordinary and partial, linear and nonlinear differential equations. HAM was presented by Liao in 1992 [91]. HAM can be employed to weak and strong nonlinear phenomena as it is independent of small physical parameter limitations.

Habibi Matin and Ghanbari [92] investigated the mixed convection of nanofluids in a vertical channel with the constant temperature walls using the model proposed by Buongiorno. The governing equations were solved using the HAM, and the results were validated with the numerical outputs. The volume fraction in the region close to the cold wall was maximum and diminished to reach a minimum value near the hot wall. In spite of the decreasing effect of Gr/Re on the volume fraction, the increase of Brinkman number led to an increase in the volume fraction across the channel.

Zhu et al. [93] analyzed the effects of second order velocity slip and nanoparticle migration for nanofluid flow between two rotating parallel plates by means of Buongiorno model employing HAM. It was concluded that heat transfer rate decreases when the ratio of Brownian motion to thermophoresis diffusion becomes larger.

Nadeem and Hussain [94] examined the nanoparticle effect on boundary layer flow of Williamson nanofluid over a stretching surface. The governing equations proposed by Buongiorno were analytically solved using HAM. It was shown that wall temperature gradient increases with an increase in Lewis number and proportion of Brownian diffusivity to thermophoretic diffusivity.

The nanofluid laminar flow and heat transfer in a vertical channel were investigated by Fakour et al. [95]. By means of a new set of similarity variables, the governing equations proposed by Buongiorno were reduced to a set of three coupled equations with an unknown constant, which were solved by the HAM. This analysis showed that nanoparticles can improve the heat transfer characteristics significantly. It was also found that Pr number plays a key role in the profiles of temperature, velocity and concentration. In addition, the buoyancy force had a negative effect on fluid motion in boundary layer, which encumbered the velocity increment near the wall, while the velocity profiles near the channel center increased with the requirement of the mass flux conservation.

Habibi Matin et al. [96] evaluated the effects of thermophoresis and Brownian motion on heat transfer of nanofluids between two rotating cylinders. The model proposed by Buongiorno was used to take into consideration the effects of particle migration. An exact solution for the velocity distribution was presented, and the HAM was employed to obtain the temperature field. The results obtained from the HAM were validated with the results of a numerical solution. It was found that with an increase in the angular velocity of

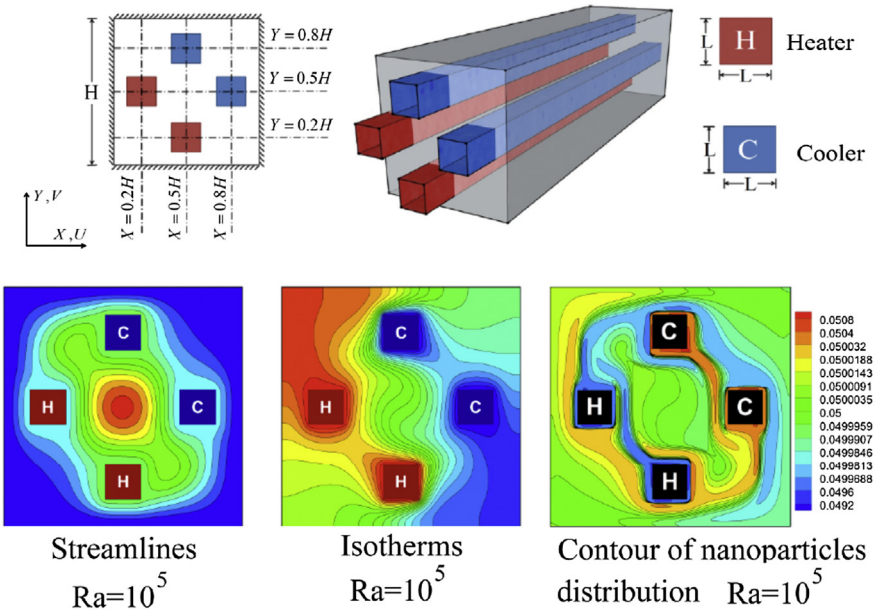

Fig. 7. The configuration of the problem and locations of heat source-sinks along with streamlines, isotherms and contour of nanoparticle distribution for $Ra = 10^5$ [90]. Reprinted with permission from Elsevier.

outer cylinder, the volume fraction in the annulus falls down. Moreover, by increasing the Brownian motion parameter or decreasing the thermophoresis parameter, the Nusselt number of the inner cylinder decreased.

Khan et al. [97] evaluated the boundary layer flow and heat transfer to Sisko nanofluid over a nonlinearly stretching sheet. The HAM was applied in order to solve the coupled nonlinear differential equations (Buongiorno model) analytically. The results showed that the temperature distribution was an increasing function of thermophoresis and Brownian motion, and concentration distribution increased with thermophoresis but decreased with Brownian motion.

To fully account for the effects of nanoparticle distribution on the continuity, momentum and energy equations, the Buongiorno model was modified by Yang et al. [98]. The Boungiorno model was modified in a way to include the nanofluid density in the conservation equations. Since the density significantly depends on nanoparticle concentration, the model can entirely consider the influences of concentration distribution. Therefore, it has the advantage of considering non-uniform profiles of thermophysical properties compared to its original form.

By considering the density, the modified equations for mass, momentum and energy can be written as below:

$$\frac{\partial \rho u_j}{\partial x_j} = 0 \tag{34}$$

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_j u_i}{\partial x_j} = -\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j}\mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{35}$$

$$\frac{\partial \rho c_p T}{\partial t} + \frac{\partial \rho c_p u_j T}{\partial x_j} = \frac{\partial}{\partial x_j} k \frac{\partial T}{\partial x_j} + \rho_p c_{p,p}\left(D_b \frac{\partial \phi}{\partial x_j} + \frac{D_T}{T}\frac{\partial T}{\partial x_j}\right)\frac{\partial T}{\partial x_j} \tag{36}$$

$$\frac{\partial \phi}{\partial t} + \frac{\partial u_j \phi}{\partial x_j} = \frac{\partial}{\partial x_j}\left(D_b \frac{\partial \phi}{\partial x_j} + \frac{D_T}{T}\frac{\partial T}{\partial x_j}\right) \tag{37}$$

In this study, Yang et al. [98] investigated forced convective heat transfer of nanofluids in an annulus. It was found that Nusselt number has optimal bulk mean volume fraction value for alumina-water nanofluids, whereas it only increases monotonously with bulk mean volume fraction for titania-water nanofluids.

The modified two-component four-equation nonhomogeneous equilibrium Buongiorno model proposed by Yang et al. [98] has been employed by some investigators. Malvandi and Ganji [99] employed the modified Buongiorno model to investigate the convective heat transfer of alumina-water nanofluid inside a microchannel in the presence of a magnetic field. Because of the microscopic roughness in microchannel and also the non-adherence of the fluid-solid interface in the presence of nanoparticle migration, known as slip condition, the Navier's slip boundary condition was considered. The results indicated that nanoparticles migrate from the heated walls towards the core region, and construct a non-uniform distribution. In addition, the ratio of the Brownian diffusivity to thermophoretic diffusivity has relatively significant effects both on the distribution of nanoparticles and heat transfer. It was further observed that the volume fraction is more uniform for smaller nanoparticles.

Moshizi et al. [100] investigated convective heat transfer and pressure drop of Al_2O_3-water nanofluid inside a concentric pipe with constant wall heat flux using modified Buongiorno model. The results revealed that nanoparticles move from the wall with higher heating energy towards the wall with lower heating energy due to the thermophoretic force. Moreover, the changes of the heat transfer coefficient in the case of heat generation was much more than that in the case of heat absorption. The geometry and coordinate systems of the annulus used in this study are shown in Fig. 8, in which the inner and outer pipe radii correspond to R_i and R_o, respectively. Moreover, η is defined as below:

$$\eta = 1 - \frac{r}{R_o} \tag{38}$$

Fig. 9 illustrates the effect of bulk mean volume fraction ϕ_B on volume fraction distribution ϕ/ϕ_B, temperature, and velocity, for a range of $\phi_B = 0$ to 0.1. In Fig. 9a, a downward trend for the volume

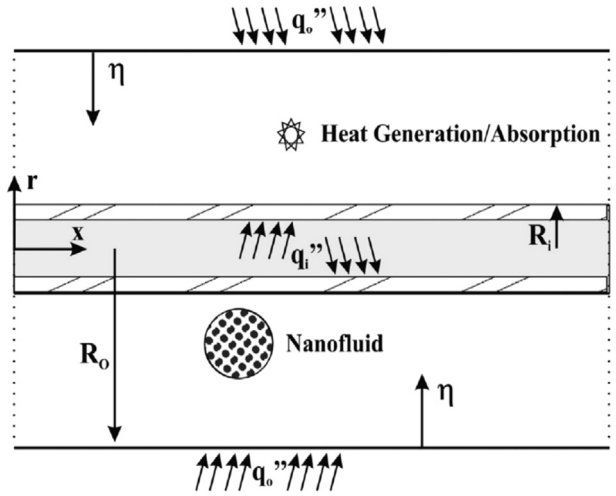

Fig. 8. The geometry and coordinate systems of the annulus [100]. Reprinted from S.A. Moshizi, A. Malvandi, D.D. Ganji, I. Pop, A two-phase theoretical study of Al_2O_3-water nanofluid flow inside a concentric pipe with heat generation/absorption, International Journal of Thermal Sciences 2014; 84: 347–357. Copyright (c) 2014 Elsevier Masson SAS. All rights reserved.

fraction in the middle region is observed by increasing ϕ_B; however, this trend is vice versa at the walls. The volume fraction increment near the outer wall is greater than that near the inner wall. An increase in ϕ_B near the outer wall leads to an increase in the viscosity in that region. Therefore, the shear stress increases on the outer wall which results in shifting the peak of the velocity towards the higher heated surface (outer wall), as observed in Fig. 9c. This increase in the dimensionless velocity near the walls decreases the dimensionless temperature there, as depicted in Fig. 9b.

Applying the modified Buongiorno model, Malvandi and Ganji [101] investigated convective heat transfer of alumina-water nanofluid inside a parallel plate channel. The upper wall was subjected to a heat flux while the bottom wall was kept adiabatic. Moreover, due to nanoparticle migration, the no-slip condition of the fluid–solid interface at the walls was abandoned in favor of a slip condition that appropriately represents the non-equilibrium region near the interface. The results indicated that the nanoparticle distribution is non-uniform such that nanoparticles move from the adiabatic wall toward the cold wall. Moreover, the anomalous heat transfer rate occurred when Brownian motion took control of nanoparticle migration (i.e. for smaller nanoparticles).

Hedayati and Domairry [102] investigated the forced convection of TiO_2–water nanofluid in a parallel plate microchannel using modified Buongiorno model. Slip condition was used at the fluid–solid interface. Considering Brownian motion and thermophoresis, the effects of nanoparticle transport on the profiles of concentration and velocity were analyzed for three different heat flux ratios of the lower to the upper walls (ε). As can be observed from Fig. 10a, when ε is unity, as thermophoresis effect is equal from both sides, a symmetric concentration profile is obtained. However, because thermophoresis increases the migration of particles in the opposite direction of the temperature gradient, when $\varepsilon < 1$ (i.e. heat flux from the upper wall is greater), the concentration value will be higher near the lower wall (Fig. 10b). In addition, it is seen that for asymmetric concentration, the peak velocity shifts towards the upper wall because of an increase in the local viscosity near the lower wall.

Although various studies have indicated that Buongiorno model presents realistic results in comparison with experimental results, however, it is worth to note that this model has been developed based on assumptions such as negligible external forces, dilute mixture, negligible viscous dissipation, as well as presence of thermal equilibrium between nanoparticles and base fluid. Thus, these assumptions should be considered when this model is used for simulation. For instance, thermal equilibrium may not be present between phases in some special conditions. Additionally, this model overlooks the effect of factors such as gravity, while several studies (for instance [64]) have emphasized the importance of gravity on nanofluid characteristics.

6. Molecular dynamics simulation

Molecular Dynamics (MD) simulation is one of the most useful atomistic modeling approaches. It is employed for simulating the molecular or atomic interactions at nanoscale, in which continuum based techniques are not applicable but a quantum mechanical method is not necessary. Indeed, it can be an effective way for evaluating the microscopic mechanisms of flow characteristics and heat transfer in nanofluids.

The basic principle of MD technique is solving molecular or

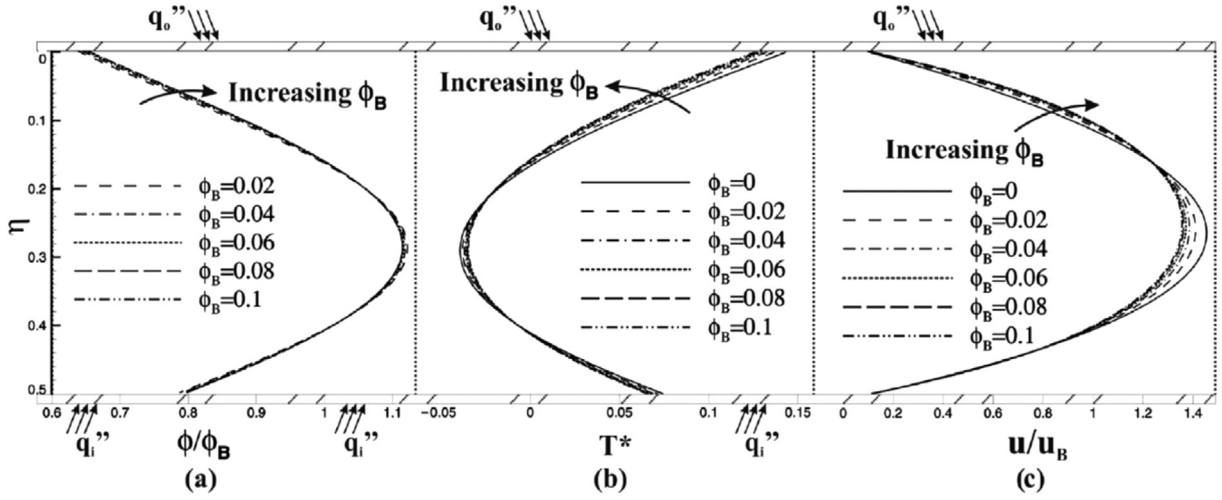

Fig. 9. The effect of bulk mean volume fraction ϕ_B on volume fraction distribution ϕ/ϕ_B, temperature, and velocity [100]. Reprinted from S.A. Moshizi, A. Malvandi, D.D. Ganji, I. Pop, A two-phase theoretical study of Al_2O_3-water nanofluid flow inside a concentric pipe with heat generation/absorption, International Journal of Thermal Sciences 2014; 84: 347–357. Copyright (c) 2014 Elsevier Masson SAS. All rights reserved.

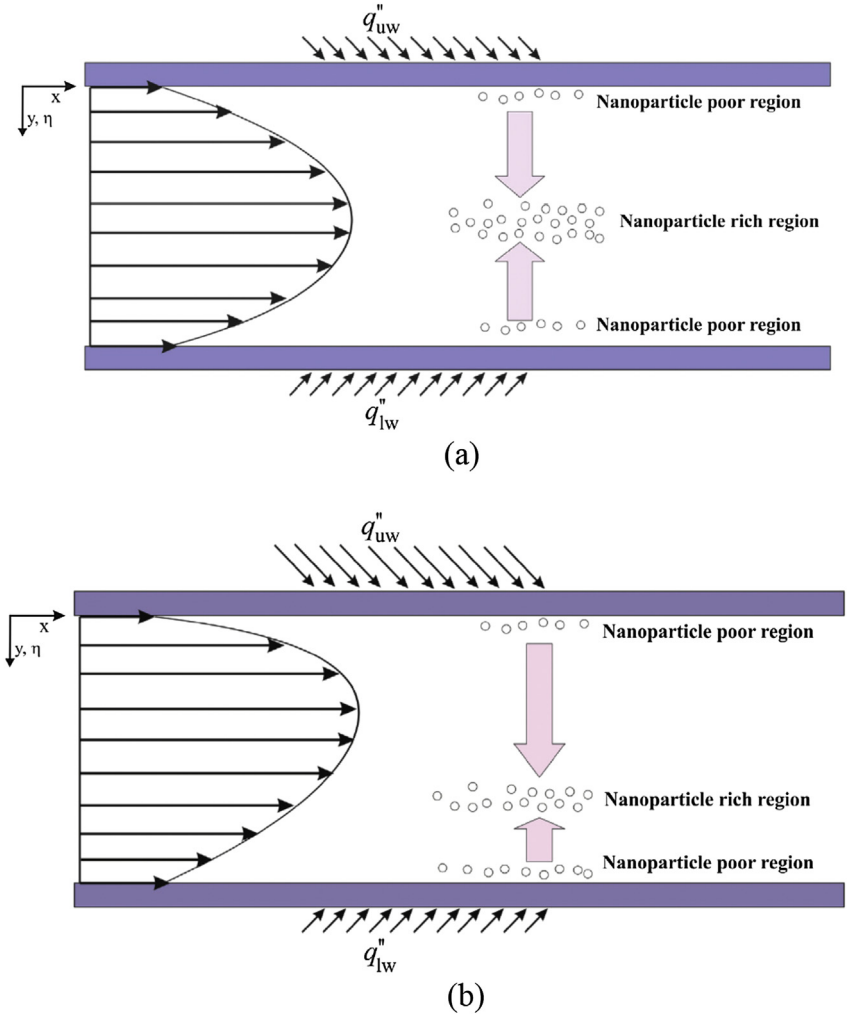

Fig. 10. Concentration distribution for different heat-flux ratios: (a) $\varepsilon = 1$, and (b) $\varepsilon < 1$ [102]. Reprinted with permission from Elsevier.

atomic Newton equations of motion considering impacts of interaction potential between molecules or atoms and external restrictions. By this approach, the time-evolving microscopic process of system is simulated, and equilibrium factors and transport characteristics can be statistically computed. Such a technique has already proved its effectiveness, at least concerning the microscopic mechanisms responsible for thermodiffusion in academic systems such as binary mixtures [103], molecular fluids [104], reactive mixtures [105], associative polymers [106], ionic systems [107], fluids in porous media [108], and polymer [109].

Many studies based on MD simulation of liquids have been presented; however, few of them have concentrated on the investigation of nanofluid characteristics. Galliero and Volz [110] proposed a new algorithm to compute single nanoparticle thermodiffusion using nonequilibrium MD simulation. It was observed that the nanoparticle migrates toward the cold area. The single particle thermal diffusion coefficient was independent of the nanoparticle size, whereas it was inversely proportional to the viscosity. The mass diffusion coefficient behavior appeared to be consistent with a Stokes-Einstein-like law.

Hu et al. [111] performed MD simulations to study friction property differences between base fluids and nanofluids in shear flow field. The results revealed that with the increase of load, liquid–solid transitions happen for both base fluids and nanofluids. The transition pressure for nanofluids was higher than that of the base fluid and the nanofluids displayed excellent friction-reducing properties when the load was high.

Cui et al. [112] investigated the movements of nanoparticles in base fluid through MD approach. By comparing the time periods of nanoparticle moving and heat diffusing, the movements of nanoparticles were found to be effective for heat transfer in nanofluids. Additionally, the effect of rotation of nanoparticles on heat transfer was proved to be comparable with translational movements. Fig. 11 illustrates MD model for nanofluid flow in the near-wall region. Along *z*-axis (the shearing velocity direction), the nanoparticles are almost flowing along with the base fluid. In the shearing velocity of 50 m/s, the average translational velocity components for nanoparticles A, B, C, were obtained to be 10 m/s, 25 m/s, 38 m/s, respectively. Along *x*- and *y*-directions, the average translational velocity components were −2 m/s ~2 m/s.

Lv et al. [113] developed impact and friction model of nanofluid for MD simulation which consists of two Cu plates and Cu–Ar nanofluid. The Lennard-Jones potential function was adopted to deal with the interactions between atoms. In the friction process, nanoparticles showed motions of rotation and translation, but affected by the interactions of nanoparticles, their rotation was trapped during the compression process. In addition, nanoparticles showed obvious aggregation phenomenon, and the effect of aggregation was more obvious with the pressure increment.

Cui et al. [114] used MD simulation to study the flow behavior of

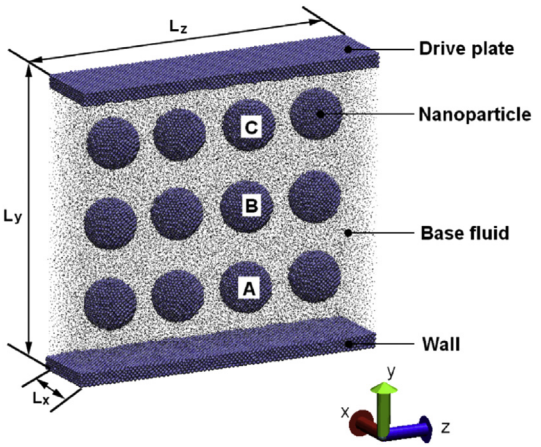

Fig. 11. MD simulation model for nanofluid flow in the near-wall region [112]. Reprinted with permission from Elsevier.

nanofluids confined in a nanochannel under different shear velocities. The nonlinearity degree of velocity profiles as well as the rotation and translation of nanoparticles intensified with the shear velocity increment. Fig. 12 depicts the nanoparticle's translations, obtained from their study, in *x*- and *y*-directions, as well as the projections under various shear velocities. In the shear velocity of 10 m/s, translations of nanoparticles were insignificant, such that the maximum displacements were 1.809 nm and 2.400 nm in *x*-direction and *y*-direction, respectively. However, by increasing the shear velocity, the displacements of nanoparticles became greater. Thereby, in the shear velocity of 100 m/s, the maximum displacements in *x*-direction and *y*-direction were 6.487 nm and 6.064 nm, respectively.

Bai et al. [115] studied the microscopic mechanism for local flow enhancement in nanofluids by MD simulation. The results revealed that the microscopic mechanism for local flow enhancement is important because the irregular movements of nanoparticles enhance momentum exchange between fluid molecules and cause disturbance of base fluid. It was found that the nanoparticles at different positions have different velocities along the flowing direction, while along the two other directions they have random translational velocities. This work suggested that the microscopic mechanism of local flow enhancement is the basis of understanding heat transfer enhancement in nanofluids.

Cui et al. [116] investigated the effects of various factors on thermal conductivity of nanofluids by MD simulation. It was found that the effective factors can be forecasted by comparing the proportion of energetic atoms containing in different nanoparticles. By tracking nanoparticles, it was illustrated that the nanoparticles move and rotate due to the bombardment of fluid molecules. The translational velocities of spherical nanoparticles were larger than those of cylindrical nanoparticle. The authors postulated that compared to displacement, the rotation of nanoparticles is more conducive to accelerating micro convection in the base fluid.

Lou and Yang [117] studied the particle size, concentration, and temperature dependence of the shear viscosity of Al_2O_3 nanofluids using equilibrium MD simulation. The nanofluid system was considered with a cubic cell of 30.0 Å in length. The Al_2O_3 particles were extracted by carving a sphere out of α-Al_2O_3 crystal. The Al and O atoms were arranged alternatively on the particle surface. The particles were then randomly placed in the cell and the rest of the space was occupied by water molecules. A snapshot of the model has been illustrated in Fig. 13. Larger viscosity was achieved for the nanofluids with smaller particle size, higher concentration, and lower temperature. The increased viscosity was attributed to the particle–water interaction.

Li et al. [118] investigated the molecular layering at liquid-solid interface in a nanofluid by equilibrium MD simulation. By tracking the positions of the nanoparticle and the liquid atoms around the nanoparticle, it was found that an absorbed slip layer is formed at the interface between the nanoparticle and liquid. Fig. 14 illustrates the position of the nanoparticle atoms and the selected liquid atoms at four different times. In order to evidently observe the trends of movements, only a small number of molecules have been shown. Through observation of the position change of the atoms in Fig. 14, it can be noticed that most liquid atoms near the nanoparticle surface always move with the nanoparticle, even though some atoms may move away from the nanoparticle surface. It was concluded that a thin layer of liquid is really formed at the interface of nanoparticle and liquid.

Some investigators employed Brownian dynamic simulation instead of MD simulation in their studies. Nanoparticles are larger than base fluid molecules and therefore, move much slower. On the other hand, the length scale of the motion of nanoparticles is much greater than that of base fluid molecules. In MD simulation, any applied method has to contain a large range of time scales. Longer

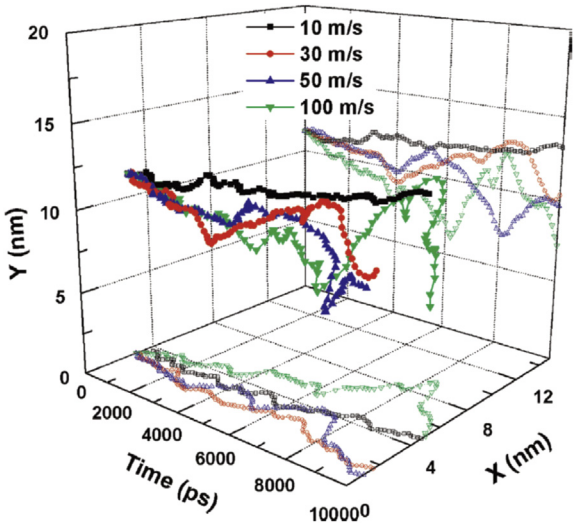

Fig. 12. Translation of nanoparticle in *x*- and *y*-directions [114].

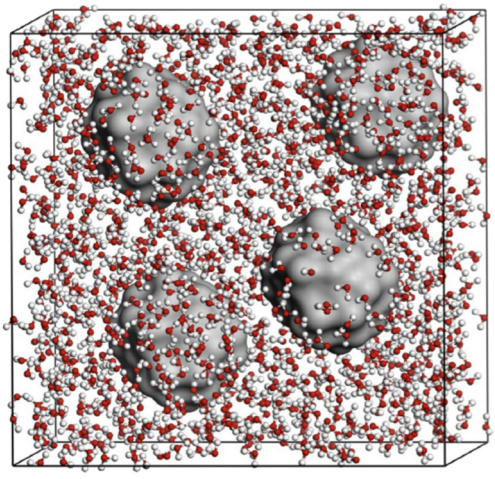

Fig. 13. A snapshot of Al_2O_3 particles (grey balls) in a cubic box filled with water [117]. Reprinted with permission from Elsevier.

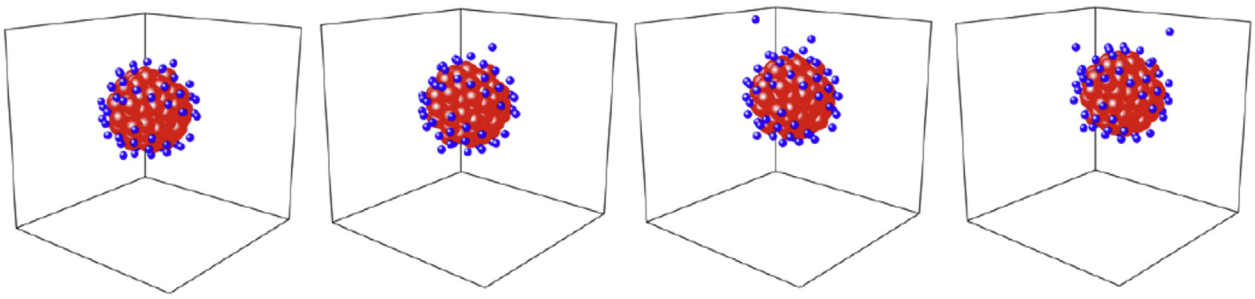

Fig. 14. The position of the nanoparticle and the liquid atoms at four different times [118]. Reprinted with permission from Elsevier.

time steps cannot be applied because it will result in an overlap of fluid particles and, therefore, erroneous results. On the other hand, smaller time steps would need a very long run to allow the complete evolution of slower mode. In Brownian dynamic simulation, however, the fluid molecules are omitted from the simulation and therefore, short-time-scale motions do not have to be calculated. In fact, the effect of hydrodynamic interactions mediated by the host fluid is considered through a spatial friction tensor. Thus, the volume of calculations decreases significantly in this method in comparison with MD approach.

Saveyn et al. [119] determined accurate particle size distribution by nanoparticle tracking analysis based on 2-D Brownian dynamics simulation. A physical model was introduced to simulate the average step length distribution during nanoparticle tracking analysis as a function of the particle size distribution and the distribution of the number of steps within the tracks. Considering only tracks of at least five steps, numerical simulation could be replaced by a normal distribution approximation. According to this model, simulation of a step length distribution allows obtaining a much more reliable estimation of the particle size distribution.

Jain et al. [120] used the Brownian dynamic simulation coupled with the Green-Kubo model in order to compute the thermal conductivity of nanofluids. The authors claimed that Brownian dynamic simulation is a better alternative to computationally expensive MD simulation. The findings displayed that Brownian motion of the particles is the most important factor in the enhancement of nanofluid thermal conductivity.

Zhang and Xiang [121] used Brownian dynamics simulations to investigate the adsorption behavior of a nanosized particle in polymer brushes. The adsorption process, the dynamic behavior of the nanoparticle, the penetration depth, the diffusion coefficient of the nanoparticle in different depths, and the forces exerted on the nanoparticle were investigated. It was shown that the behavior of the force curve is a result of the competition of attractive interaction and steric repulsion between the nanoparticle and polymer molecules.

Gupta and Kumar [122] carried out Brownian dynamic simulation of a nanofluid in which the interparticle potential was determined based on Debye length and surface interaction of the fluid-solid. It was shown that Brownian motion can increase nanofluid thermal conductivity by 6% primarily due to random walk motion and not only through diffusion.

Generally speaking, during recent years, MD simulation has been implemented to predict thermal characteristics of nanofluids, such that it has provided comprehensive information about heat transfer mechanisms at nanoscale. The MD technique is a powerful method that simulates the behavior of materials and, through that, the trajectory and movements of atoms, molecules, and nanoparticles can be determined. Furthermore, it can predict thermophysical properties by assuming an appropriate potential between particles. However, there are some limitations in MD technique such as number of particles and time of simulation. In fact, it is unrealistic to deploy full MD simulations to investigate flows at micro- and nano-scales as computational time and volume increase considerably. It should be noted that inaccuracy of the continuum description is frequently confined to partial domains, such as fluid–fluid or fluid–solid interfaces. Therefore, it is not necessary to use MD method in all regions and it is appropriate to apply hybrid techniques that combine continuum fluid dynamics and MD simulation. It is thus necessary to pay greater attention to hybrid techniques for nanofluids in future studies.

The other issue that should be noted is that in simulations performed using MD method, researchers have often modeled the problems using many simplifying assumptions. Indeed, the interaction of nanoparticle surface and fluid molecules is really complex, and is profoundly dependent on nanoparticle surface, surface energy content resolution, orientation, texture and dynamics of nanoparticle. The simplified models commonly assume a circular nanoparticle in 2D simulation or a spherical nanoparticle in 3D simulation. It is also assumed the linear transfer of energy through a nanoparticle with uniform surface energy content, ignoring the effects of surface texture and spin. Therefore, the findings obtained from MD simulation existing in the literature may have been influenced by these assumptions. As a conclusion, there is an essential need to apply types of MD simulations in which the fewest number of simplifying assumptions are applied on one hand, and the volume of computations is not increased significantly on the other hand. In future, these models can provide detailed knowledge about heat transport mechanisms in nanofluids.

7. The viscosity of nanofluids considering particle migration

Due to modification in concentration distribution, particle migration can cause changes in viscosity profile of nanofluids. In some studies, the effect of particle migration on viscosity has been evaluated.

Malvandi and Ganji [123] investigated mixed convective heat transfer of a nanofluid inside a vertical microchannel considering nanoparticle migration. Their results revealed that nanoparticles move from the heated walls toward the core region of the channel and construct a non-uniform nanoparticle distribution. This non-uniform distribution reduced the viscosity near the wall and increased it in the core region of the channel.

Ryzhkov and Minakov [124] evaluated laminar convective heat transfer of water–alumina nanofluid in a circular tube with uniform heat flux on the basis of two-component model, which took into account nanoparticle transport. It was shown that the reduction of volume fraction in the boundary layer, caused by particle migration, decreases the viscosity of nanofluid near the wall. The viscosity reduction led to the increase of velocity adjacent to the wall.

Bahiraei et al. [125] used the thermal dispersion model to

simulate heat transfer of water–Al_2O_3 nanofluid considering the effects of particle migration. The nanofluid viscosity in their study showed a non-uniform distribution due to the non-uniform concentration where this non-uniformity was intensified by raising the Reynolds number. Regarding the significant importance of the viscosity value near the wall and its effect on pressure drop, the authors claimed that lower viscosity near the wall can affect the pressure drop positively in comparison with uniform concentration.

Masoumi et al. [126] developed a theoretical model for nanofluid viscosity with the consideration of the Brownian motion. The model (Eq. (39)) was further confirmed by Kole and Dey [127] and Peyghambarzadeh et al. [128].

$$\mu = \mu_f + \frac{\rho_p V_b d_p^2}{72C\delta} \tag{39}$$

where δ and V_b represent distance between the particles and the Brownian velocity of the nanoparticles, respectively. They can be obtained from equations below:

$$V_b = \frac{1}{d_p}\sqrt{\frac{18k_bT}{\pi\rho_p d_p}} \tag{40}$$

$$\delta = \sqrt[3]{\frac{\pi}{6\phi}}d_p \tag{41}$$

C is defined as:

$$C = \frac{1}{\mu_f}\left[(c_1 d_p + c_2)\phi + (c_3 d_p + c_4)\right] \tag{42}$$

where

$c_1 = -0.000001133$, $c_2 = -0.000002771$, $c_3 = 0.00000009$, $c_4 = -0.000000393$.

8. The role of particle migration due to Brownian motion in thermal conductivity

Many authors [129–131] have claimed that the great thermal conductivity of nanofluids is due to hydrodynamic effects of Brownian motion of nanoparticles. These authors argued that each Brownian particle causes a long velocity field in the surrounding fluid, similar to that is present around a particle moving with a constant velocity, that declines approximately as the inverse of distance from center of particle. In fact, since the mass of fluid molecules is very small in comparison to that of nanoparticles, the impacts of individual molecular collisions on the particles are minor. However, the number of molecular impacts per unit time is very large and their aggregate impact can be significant on the motion of fine particles.

In most investigations, the observed enhancements for thermal conductivity of nanofluids are far beyond the predictions of the effective medium approximation. Therefore, researchers have tried to renovate the effective medium approximation by including other mechanisms especially Brownian motion of nanoparticles.

Although some researchers indicated that the effect of Brownian motion on thermal conductivity of nanofluids is minor [132–134], several correlations for thermal conductivity of nanofluids based on Brownian motion have been developed [129,135,136].

Many studies have emphasized the significant role of Brownian motion. Azizian et al. [137] performed a set of experiments on titanium dioxide-water nanofluids to consider the effect of material on Brownian motion. The findings indicated that Brownian motion is the dominant mechanism responsible for the observed enhancements in thermal conductivity. It was observed that the relation between the particle loading and thermal conductivity follows a parabolic profile.

Considering Brownian motion of nanoparticles, Xiao et al. [138] developed an analytical model for thermal conductivity of nanofluids. The formula of thermal conductivity was given by taking into account the fractal distribution of nanoparticles. It was found that nanofluid thermal conductivity for smaller particles was larger than that for bigger particles.

Wang et al. [139] suggested a model considering the roles of Brownian motion and nanolayer at the particle-fluid interface to predict the thermal conductivity and its temperature dependence of nanofluids. It was shown that Brownian movement and temperature dependence of viscosity are responsible for thermal conductivity enhancement.

Some researchers have evaluated the effect of micro-convection due to Brownian motion on thermal conductivity in nanofluids. Mallick et al. [131] developed a new model for thermal conductivity of nanofluids by employing Prandtl, Reynolds and Brinkman numbers, representing the effects of micro-convection, localized turbulence and the ratio of heat transfer by diffusion to conduction for particle and fluids. Assessment of this new model by comparing the predicted results against experimental data showed that the model is within 5% accuracy for a wide range of data.

Nabi and Shirani [140] considered Brownian motion induced micro-convection to model the thermal conductivity of nanofluids. The authors modified the conventional equations, and derived an equation for thermal conductivity, which is a combination of Brownian motion thermal conductivity for both aggregates and single particles.

Prasher et al. [141] showed through an order-of-magnitude analysis that enhancement in thermal conductivity of nanofluids is mainly due to the convection caused by Brownian movement of nanoparticles. They also introduced a convective-conductive model which accurately captures the effects of particle size, choice of base liquid, thermal interfacial resistance between particles and liquid, temperature, and so forth. Their model was a combination of the Maxwell-Garnett conduction model and convection caused by Brownian movement. The model showed that the lighter the nanoparticles, the greater the convection effect in the liquid, regardless of nanoparticle thermal conductivity.

Some other researchers claimed that Brownian motion does not exert a great effect on thermal conductivity of nanofluids. Evans et al. [134] used a kinetic theory for suspensions of nanoparticles and demonstrated that the hydrodynamic effects associated with Brownian motion have only a minor effect on thermal conductivity of nanofluids. They supported their argument with the results of MD simulations of a model nanofluid. It was found that heat transport via a conduction mechanism is much faster than nanoparticle motion. The authors indicated that the thermal conductivity of a nanofluid with well dispersed nanoparticles is well described by the effective medium theory.

The effect of Brownian diffusion on nonequilibrium heat conduction in a nanofluid layer with periodic heat flux on one side and specified temperature on the other side was numerically investigated by Zhang et al. [142]. The results showed that Brownian diffusion only affects nanoparticle temperature, but its effect on heat transfer is negligible.

Shima et al. [143] investigated the role of micro-convection induced by Brownian motion of nanoparticles on thermal conductivity of nanofluids. Their findings demonstrate that micro-convection is not the key mechanism responsible for thermal

conductivity enhancement in nanofluids whereas aggregation has a more prominent influence.

Loulijat et al. [132] showed the influence of the solid–solid inter–atomic potential type on the thermal conductivity of the Ar–Cu nanofluid. They concluded that the Brownian motion of the Cu nanoparticles has less importance role in the thermal conductivity enhancement.

In general, the existing contradictions in this field show that the role of Brownian motion on heat transfer improvement is still being debated and requires serious studies. The investigations about the effect of Brownian motion on the characteristics of nanofluids have resulted in an open inconsistency. The lack of experimental data on the issue contributes to the uncertainty surrounding the problem. The disagreement may only be solved through definitive experiments and analyses, which consider not only the movement of particles, but also the induced motion of the fluid and all the transient effects in the fluid. Moreover, accurate numerical simulations that consider all the relevant variables would be helpful to resolve this open question.

9. The effect of stabilizers on the particle migration and stability of nanofluids

In nanofluids, nanoparticles have the tendency to aggregate due to the great surface area and surface activity. The agglomeration of nanoparticles causes not only the settlement and clogging of ducts but also reducing the thermal conductivity. Therefore, the investigation of stability is an important issue that affects the properties of nanofluids, and it is essential to assess effective factors on the dispersion stability of nanofluids. The concept of stability is that the particles do not aggregate at a substantial rate. Generally, the aggregation rate is evaluated by the frequency of collisions and the possibility of cohesion during collision.

One important method to improve the stability of nanoparticles in liquids is the implementation of surfactants. Surfactants can significantly affect the surface characteristics. They include a hydrophobic tail portion, usually a long-chain hydrocarbon, and a hydrophilic polar head group. Surfactants are implemented to enhance the contact of two materials, occasionally known as wettability.

Derjaguin, Verway, Landau, and Overbeek (DVLO) presented a theory which deals with colloidal stability [144]. DLVO theory indicates that the stability of a particle in suspension is determined by the sum of van der Waals attractive and electrical double layer repulsive forces that exist between particles. If the attractive force is greater than the repulsive force, the two particles will collide, and thus, the suspension will not be stable. If the particles have an enough high repulsion, the suspension will have a stable state.

In relation to the kinds of repulsion, the fundamental mechanisms that affect stability of nanofluids are divided into two types, one is steric repulsion, and another is electrostatic repulsion (see Fig. 15). About steric stabilization, polymers are involved into the suspension, and they will adsorb onto the particles surface, developing an additional steric repulsive force. About electrostatic stabilization, surface charge will be created using one or more of the mechanisms below:

- Isomorphic substitution of ions
- Preferential adsorption of ions
- Accumulation or depletion of electrons at the surface
- Dissociation of surface charged species
- Physical adsorption of charged species onto the surface.

Several studies have been performed about effects of different surfactants on stability of nanofluids. Xia et al. [146] prepared

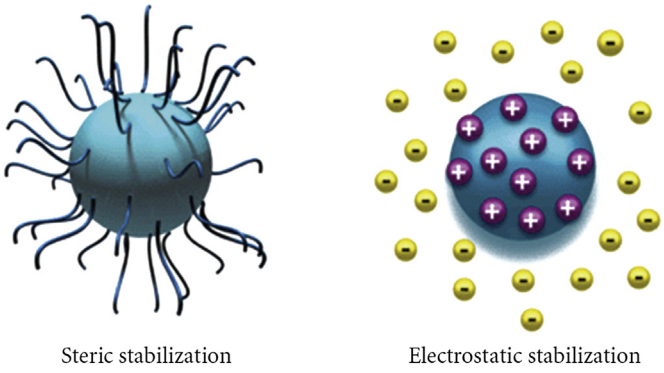

Fig. 15. Types of colloidal stabilization [145].

Al_2O_3–water nanofluids of different volume concentrations with various surfactant mass fractions. Effects of two kinds of surfactants, sodium dodecyl sulphate (SDS) and polyvinylpyrrolidone (PVP), on the stability of the nanofluid were analyzed. The results revealed that surfactant plays an important role in dispersing the nanoparticles into the base fluid and improving the stability of the nanofluid. Non-ionic surfactant PVP showed better positive effects than anionic surfactant SDS on the dispersion and stability of the nanofluid.

Yang et al. [147] prepared a nanofluid by adding Al_2O_3 nanoparticles with sodium dodecyl benzene sulfonate (SDBS) in the ammonia-water solution. The dispersion stability of the nanofluid in different mass fractions of surfactant was investigated by the light absorbency ratio index methods. The results showed that with the increasing mass fraction of surfactant, the stability of the nanofluid is exacerbated firstly, then is improved, and then is exacerbated again. In this study, the surfactant mass fractions required in the preparation of the nanofluid were calculated by simplifying the dispersion models, and the results were in agreement with experimental results.

10. Investigating nanoparticle migration in some fascinating topics

Some research studies have investigated the role of particle migration in several interesting topics that will be introduced in the following sections.

10.1. Nanoparticle migration in the presence of a magnetic field

Particle motion and heat transfer rate in nanofluids can be controlled applying magnetic fields. This has attracted many researchers' attention. Some of these researchers have studied nanoparticle migration under the effect of magnetic fields.

Wu et al. [148] applied a magnetic field using a permanent magnet and showed increased migration of nanoparticles into the fluid channel. Particle velocities were estimated from magnetic and hydrodynamic interaction forces. It was shown how particle separation is affected by Peclet number, channel length to width ratio, and magnetic field strength. The main finding showed that in the presence of a magnetic field, particle migration is much greater than that predicted by theory.

Sheikholeslami et al. [149] investigated MHD effect on nanofluid natural convection in an enclosure. The transport equations took into account the effects of Brownian motion and thermophoresis. The Navier-Stokes equations in vorticity-stream function form were used to simulate the flow pattern, isotherms and concentration. The results indicated that Nusselt number is an increasing function

of buoyancy ratio number but it is a decreasing function of Lewis number. Moreover, the concentration boundary layer thickness near inner wall increased by Hartmann number increment.

Bahiraei and Hangi [150] investigated numerically the performance of water based Mn–Zn ferrite magnetic nanofluid in a double–pipe heat exchanger under quadrupole magnetic field (Fig. 16). The results showed that application of the magnetic field makes the distribution of particles more uniform and this uniformity increases by increasing the distance from the tube inlet. Indeed, in contrast to shear rate that leads particles to central regions, the magnetic force makes particles to become absorbed to the wall. In addition, increasing each of the parameters of concentration, particle size and magnitude of the magnetic field led to a greater pressure drop and also higher heat transfer improvement. Moreover, at higher Reynolds numbers, the effect of magnetic force was diminished.

The boundary layer flow and heat transfer over a permeable stretching sheet due to a nanofluid with the magnetic field effects, slip boundary condition and thermal radiation were investigated by Ibrahim and Shankar [151]. The results revealed that the local Nusselt number decreases by increasing both Brownian motion and thermophoresis.

Malvandi et al. [152] studied convective heat transfer of a nanofluid between two concentric cylinders in the presence of a radial magnetic field. To consider the effects of boundary condition on nanoparticle migration, two distinctive cases including constant heat flux at the outer wall and adiabatic inner wall (case A), and constant heat flux at the inner wall with adiabatic outer wall (case B) were considered. Their results indicated that due to thermophoretic force, the concentration of nanoparticles was greater at the adiabatic wall for the case A. Moreover, inducing the magnetic field, heat transfer rate was increased for the case A which had a decreasing effect on the case B.

Bahiraei et al. [153] assessed flow and heat transfer characteristics of the water-$MnZnFe_2O_4$ nanofluid through an annulus under the effect of a non-uniform magnetic field. The concentration distribution was found to be non-uniform whose value was lower near the walls. It was found that the velocity profile becomes flatter at the annulus cross section because of particle migration due to applying the magnetic field. Furthermore, the effect of increasing magnitude of the magnetic field gradient on heat transfer and pressure drop was more significant for larger particles.

The above studies show clearly that nanoparticle migration under the influence of a magnetic field can have great effects on characteristics of nanofluids. However, further studies are required for better characterizing nanofluid features in the presence of magnetic fields and understanding the effect of particle migration in these conditions.

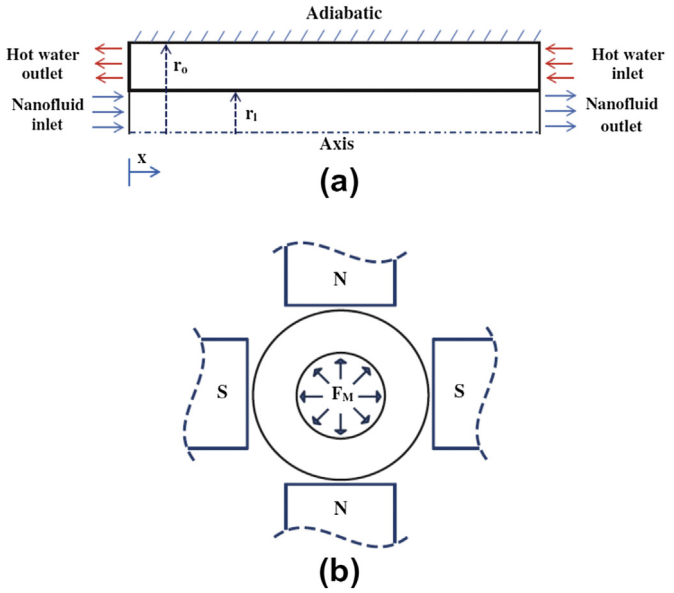

Fig. 16. The configuration of problem: (a) longitudinal cross section and (b) transverse cross section [150]. Reprinted with permission from Elsevier.

10.2. Nanoparticle migration in microchannels

Recently, many investigators have shown interest in small scale flows and many efforts have been carried out in minimization of scales for improvement of the efficacy of instruments. The usage of microchannels is one of the most promising methods for heat transfer enhancement. By increasing the use of miniaturized instruments such as microchannels, characterization of the behavior of such flows has become more essential. Some studies have been conducted to improve our understanding about nanoparticle migration in nanofluids through microchannels.

Afshar et al. [154] solved the Navier–Stokes and energy equations in a microchannel, and temperature and velocity profiles were evaluated. Dispersion of nanoparticles due to drag, Brownian, gravity, and Saffman lift forces was studied. Microchannel dimensions were comparable to mean free path of the molecules; thus, the carrier phase was considered to be in slip flow regime. Because of the dilution, particle collision was neglected and it was assumed that the dispersed phase does not affect the velocity and temperature of the carrier phase. The trajectories of 50 nm particles in various pressure differences were depicted. It was shown that particles travel in different paths because of the random nature of Brownian force. In addition, the residence time of particles changed by variation of pressure difference between two sides of the microchannel, and in the low pressure differences, nanoparticles did not follow the stream lines anymore and the effect of Brownian motion was completely significant (see Fig. 17).

Malvandi and Ganji [155] investigated effects of nanoparticle migration on convective heat transfer of a nanofluid in microchannels. The walls were subjected to different heat fluxes, and because of non-adherence of the fluid-solid interface caused by the microscopic roughness, Navier's slip was applied at the surfaces. It was revealed that nanoparticles eject themselves from heated walls, construct a depleted region, and accumulate in the core region, but more likely to accumulate near the wall with lower heat flux. Moreover, the non-uniform nanoparticle distribution caused velocities to move toward the wall with a greater heat flux and increased heat transfer rate there.

Hedayati and Domairry [156] evaluated the effects of nanoparticle migration on mixed convection of titania/water nanofluid inside a vertical microchannel via Runge–Kutta–Fehlberg method. Because of small dimensions of microchannels, a linear slip condition was assumed at the boundaries. It was found that the asymmetric boundary condition affects the direction of nanoparticle migration and distorts the symmetry of the velocity and temperature profiles.

10.3. Nanoparticle migration in boiling of nanorefrigerants

Migration features of nanoparticles in the pool boiling of nanorefrigerants are fundamental knowledge when dealing with nanorefrigerants in refrigeration systems. When applying nanorefrigerants, one important subject to be considered is the migration of nanoparticles during the boiling process. This knowledge will help to clarify how distribution of nanoparticles affects the

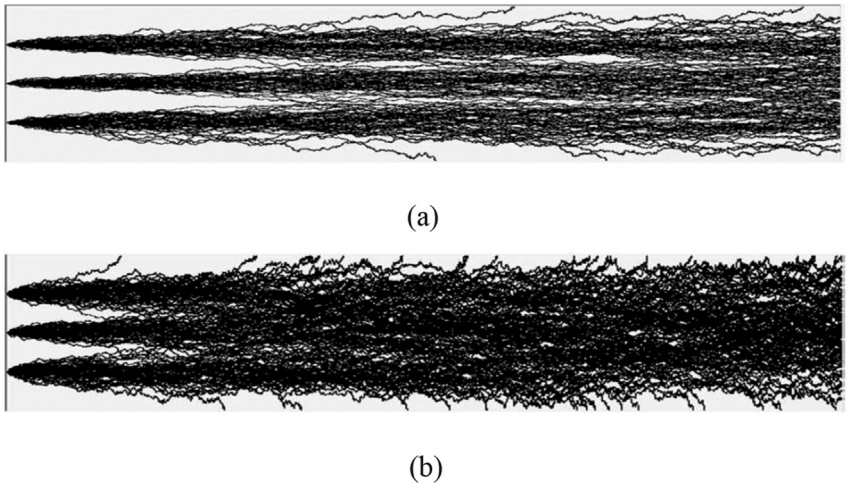

Fig. 17. Dispersion of 50 nm particles for different values of pressure drop; a) 100 kPa, b) 30 kPa [154]. Reprinted with permission from Elsevier.

cycle behavior of a refrigeration system.

Some studies have dealt with nanoparticle migration in boiling of nanorefrigerants. Ding et al. [157], for the first time, investigated the major factors influencing the characteristics of CuO nanoparticle migration during pool boiling of R113 refrigerant. A comparison between pure nanofluid and nanofluid-oil mixture was made. They demonstrated that the original mass of nanoparticles and the mass of nanorefrigerant affect migration rate.

Influences of refrigerant-based nanofluid composition and heating condition on migration of nanoparticles during pool boiling were experimentally investigated by Peng et al. [158]. The results showed that migration ratio increases with a decrease in particle density, particle size, dynamic viscosity of refrigerant, mass fraction of lubricating oil, and heat flux. It was outlined that due to the exclusion of parameters considering nanofluid composition and heating conditions, the model of Ding et al. [157] would not be able to predict migration features. Hence, Peng et al. [47] proposed a more inclusive model that was claimed to be 90% accurate in accordance with experimental data.

Mahbubul et al. [159] focused on characterizing the migration properties of TiO_2 nanoparticles during the boiling process of R141b refrigerant. Increasing the heat flux and initial mass, and insulating the container would yield augmentation in the amount of migrated particles from the liquid refrigerant. The results showed that the migrated mass increases with augmenting the initial mass of nanoparticles as well as the heat flux. Besides, particle departure from the liquid to vapor augmented by increasing the lubricating oil concentration and adding insulation to the container. However, the migration of nanoparticles decreased with an increase in initial liquid level height and boiling vessel size.

11. Conclusion

The current study attempted to present a comprehensive review of investigations performed on the field of nanofluids considering particle migration. According to this review, application of nanofluids has been changing from an innovative concept into a reality in the past years. However, the results obtained from different research groups are inconsistent; overlooking the effect of nanoparticle migration in previous studies is one of the reasons for this inconsistency. By the time the findings of various studies differ, no comprehensive model for characteristics of nanofluids can be reached. However, to bridge the research gaps, the following critical research directions can be postulated:

1) Newer numerical methods such as lattice-Boltzmann method can hopefully present insight into distribution and migration of nanoparticles.
2) While the suggested particle migration models yield useful information for laminar flow, it is inadequate to clarify the transition to turbulence and the flow behavior under turbulent conditions. A new approach should, therefore, be sought to provide insight into the flow physics in transition and turbulent regimes. Studies should focus on nanoparticle behaviors and even interactions between eddies and nanoparticles. Based on these assessments, more practical models can be derived for nanofluids.
3) Although the results of MD method are to some extent correct and reliable, it can only afford simulation for a very small domain. The reason is that solving the Newton's equation of motion for a large system is quite time-consuming. Therefore, it is appropriate to employ hybrid techniques that combine continuum fluid dynamics and MD simulation. In addition, researchers have employed many simplifying assumptions in MD method which may have affected the existing findings in the literature. As a consequence, there is an essential need to apply types of MD simulations in which fewest number of such simplifying assumptions have been employed on one hand, and the volume of calculations does not increase much on the other hand. Such models can provide, in future, the detailed knowledge about flow and thermal mechanisms in nanofluids.
4) The migration of nanoparticles due to Brownian motion or thermophoresis should also be evaluated by experiments.
5) Although Buongiorno model can present rational results in comparison with experimental results, it should be noted that this model has been developed using assumptions such as negligible external forces, dilute mixture, negligible viscous dissipation, as well as existing thermal equilibrium between nanoparticles and base fluid. Therefore, such assumptions should be taken into consideration when such simulations are employed.
6) In the Eulerian-Lagrangian approach, the formats of most of the forces that have been utilized for nanofluids have been established for relatively large particles. They may not be applicable to nanoparticles, due to for example the rarefaction, and it is not obvious currently how these correlations are corrected for nanoparticles. Models are required to be

developed specifically for nanofluids for more accurate studies in the future.

7) In the analytical modeling of particle migration, it is necessary to employ more comprehensive models considering factors such as entrance effects, dynamics of particles, particle–wall interactions, and so forth.
8) Particle migration for non-Newtonian nanofluids, which can be different from Newtonian nanofluids due to the nonlinear correlation of stress and strain rate, should be studied.
9) According to the Ref. [160], entropy generation in nanofluids is affected by velocity and temperature fields. Therefore, as nanoparticle migration in nanofluids causes changes in velocity and temperature profiles [40,44], it can affect the entropy generation rate and it needs to be taken into consideration in future studies for analyses of the second law of thermodynamics in this area.
10) The role of Brownian motion is still being debated and requires serious investigations. The disagreements observed in various surveys may only be solved with definitive experiments and analyses, which consider not only particle movements, but also the induced motion of fluid and all transient effects.
11) Applying hybrid nanofluids is a new challenge and opportunity. Hybrid nanofluids, prepared from mixing various types of nanoparticles, can be employed in the future as promising nanofluids for enhancement of heat transfer. Particle migration in them can be different in comparison with conventional nanofluids and this needs to be investigated. It can open the road for development of miscellaneous nanofluids with many extraordinary applications.

Once such details are obtained about the above important issues, more confidence can be gained in conducting applied investigations of nanofluids in various areas with more efficiency.

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